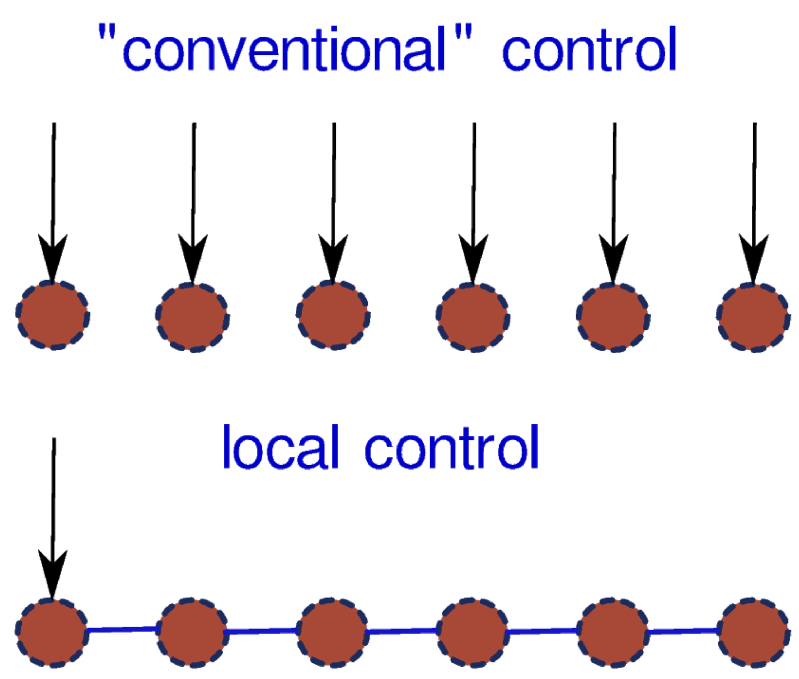
Quantum control of interacting qubits

Vladimir M. Stojanović[1], Rahel Heule[1], Daniel Burgarth[2,3], and C. Bruder[1]

[1] Department of Physics, University of Basel, Switzerland
[2] Institute for Mathematical Sciences, Imperial College London, United Kingdom
[3] QOLS, The Blackett Laboratory, Imperial College London, United Kingdom

Local control approach for interacting systems



Complete controllability:

Acting locally on one of the end spins of an XXZ-Heisenberg spin chain ensures complete controllability.
[see, e.g., D. Burgarth *et al.*, PRA **79**, 060305(R) (2009)]

XXZ Heisenberg spin-1/2 chain

Total Hamiltonian: $\boldsymbol{H(t) = H_0 + H_c(t)}$

XXZ Heisenberg coupling with anisotropy Δ (chain length N_s):

$$\boldsymbol{H_0 = J\sum_{i=1}^{N_s-1}(S_{ix}S_{i+1,x} + S_{iy}S_{i+1,y} + \Delta S_{iz}S_{i+1,z})}$$

Zeeman-like control part (control fields acting on the first spin only):
$\boldsymbol{H_c(t) = h_x(t)S_{1x} + h_y(t)S_{1y}}$

Dynamical Lie algebra generated by $\boldsymbol{iH_0}$, $\boldsymbol{iS_{1x}}$, $\boldsymbol{iS_{1y}}$ has dimension $\boldsymbol{d^2-1}$ ($d = 2^{N_s}$) $\Rightarrow$ **complete controllability**

Operator control

Realize a desired unitary transformation (target quantum gate) $\boldsymbol{U_{\text{target}}}$ at a time $t = t_f$ (more general than state control).

Control objectives (target gates)

One-qubit gate:
$\boldsymbol{X}$ (flip on the last qubit)

Entangling two-qubit gates:
CNOT and $\sqrt{\textbf{SWAP}}$ (on the last two qubits)

Gate fidelity is defined as

$$\boldsymbol{F(t_f) = \frac{1}{d}\left|\mathrm{tr}\left[U^\dagger(t_f)U_{\text{target}}\right]\right|}$$

$\boldsymbol{U(t_f)}$: time-evolution operator at $t = t_f$

THREE-SPIN SYSTEM

Control fields implementing a flip of the third qubit

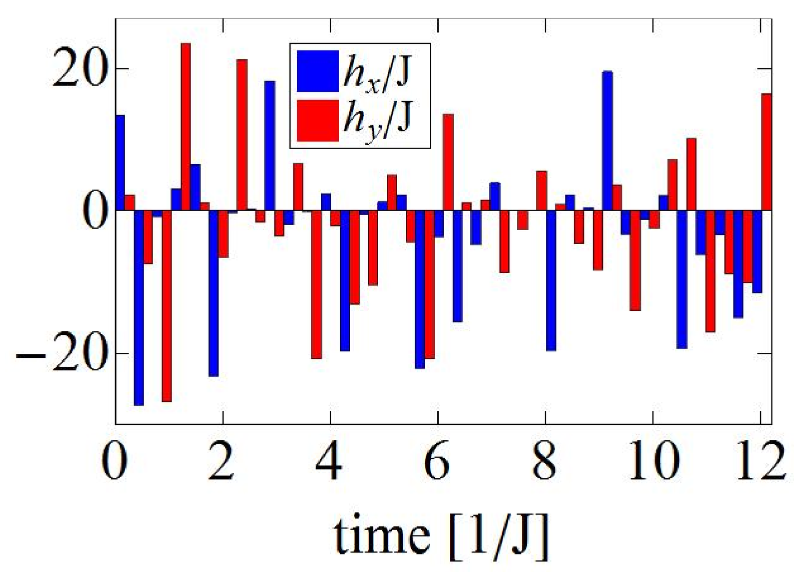
Piecewise-constant control fields: N_t pulses of length T (total evolution time $t_f = N_tT$)
Maximization of fidelity with respect to the N_t field amplitudes $\Rightarrow$ **optimal control sequences (gate error: $< 10^{-3}$)**
Estimate of minimal gate times

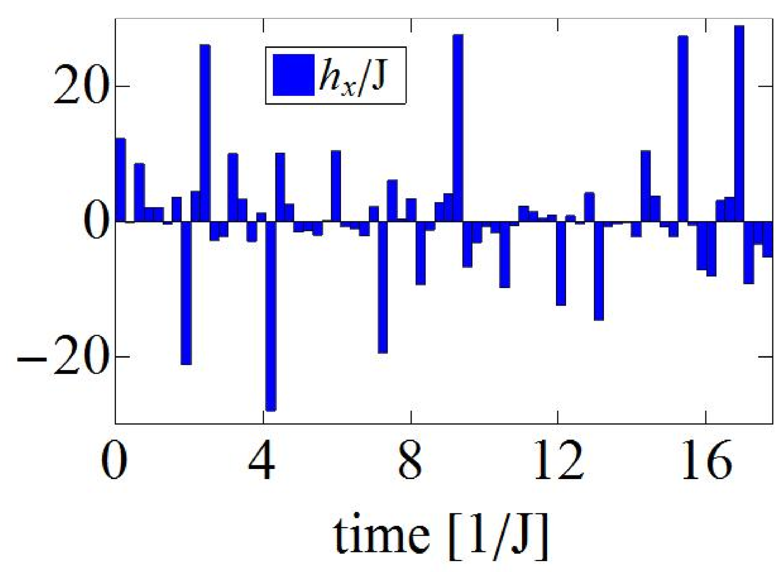
Alternate x and y pulses:
$\boldsymbol{h_{x,j}}$, $\boldsymbol{h_{y,j}}$ ($j = 1, ..., N_t/2$)

Lie completion: $[S_{1x}, S_{1y}] = iS_{1z}$

$\Rightarrow$

System is completely controllable, i.e., the dimension of the dynamical Lie algebra is 63.





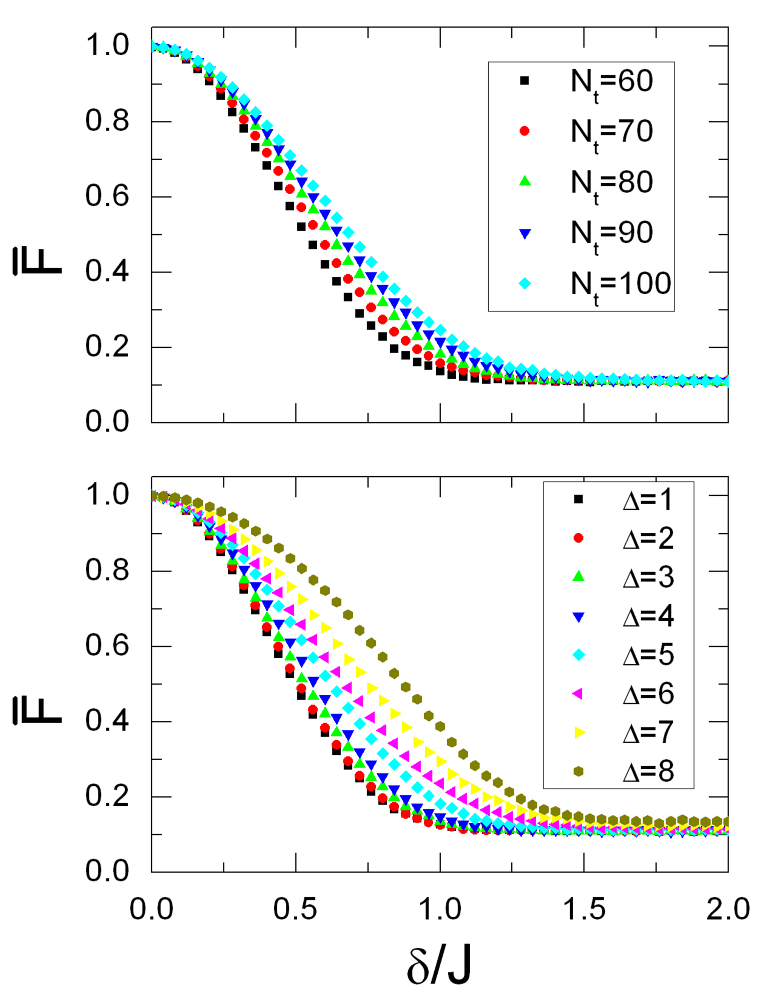
Controls in x direction only:
$\boldsymbol{h_{x,j}}$ ($j = 1, ..., N_t$)

Dimension of the dynamical Lie algebra: 30 for $\Delta \neq 1$ and 18 for $\Delta = 1$

Only specific gates can be realized, such as $\boldsymbol{X}$ and $\sqrt{\textbf{SWAP}}$ gates.

Robustness of fidelity to random errors in the control fields

$\boldsymbol{\bar{F}}$: fidelity averaged over random realizations
$\boldsymbol{\delta}$: half-width of uniform random distribution
Gate implementation using **x and y controls**



$\sqrt{\textbf{SWAP}}$ on the second and third qubit
($\Delta = 1.2$, $t_f = 60J^{-1}$)

Fixed $\boldsymbol{t_f}$: Fidelity less sensitive to random errors for faster switching (larger $\boldsymbol{N_t}$).

CNOT on the second and third qubit
($N_t = 70$, $T = 1J^{-1}$)

Robustness depending on $\boldsymbol{\Delta}$: Larger $\boldsymbol{\Delta}$ yield higher fidelity.

Universal saturation at a value of $\boldsymbol{1/d = 0.125}$ for any $\boldsymbol{t_f}$
(in the completely-controllable case)
[see R. Heule *et al.*, PRA **82**, 052333 (2010), and arXiv:1010.5715 for more details]

Spectral filtering (smoothing) of control fields

Frequency-filtered control fields:

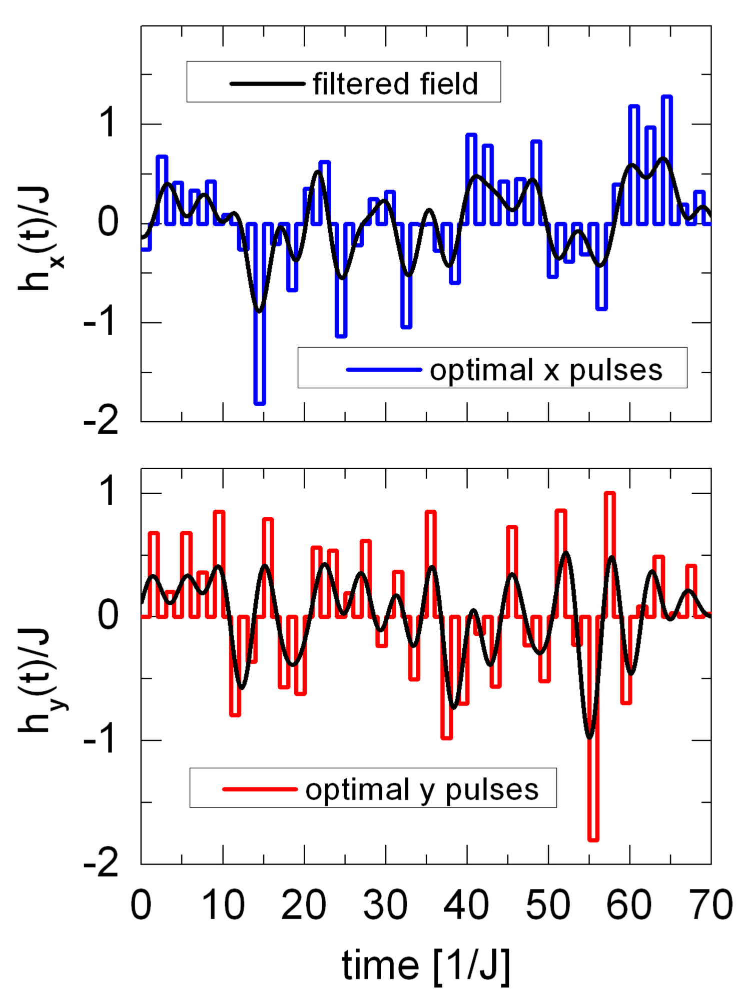
$$\tilde{h}_j(t) = \mathcal{F}^{-1}\left[f(\omega)\mathcal{F}[h_j(t)]\right] \quad (j = x, y)$$

Ideal low-pass filter:

$$f(\omega) = \theta(\omega + \omega_0) - \theta(\omega - \omega_0)$$

Example: implementation of **CNOT** on the second and third qubit with a gate error of 10^{-8}

Filtered control field retains a fidelity of 0.9.
(with a cut-off $\omega_0/J = \pi/2$)



Conclusions and Outlook

- Minimal gate times strongly depend on Δ and grow rapidly with the chain length.
- Relatively high gate fidelities possible in the presence of random errors and for smoothened control fields.
- Generalizations: spins $s > 1/2$, more complex control pulses, open-system scenario.