From quantum control to one-way quantum computing in interacting qubit arrays

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Outline of the talk

- **Introduction**

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- **Quantum control in qubit arrays with "always-on" interactions**

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- **Conclusions and Outlook**

Coupling between qubits

$$H_{\rm int} = \sum_{i<j} \sum_{\alpha,\beta} J_{ij}^{\alpha\beta} \sigma_i^{\alpha} \sigma_j^{\beta} \quad (\,\alpha, \beta = x, y, z\,)$$

qubit-qubit interaction	qubit system
Ising	charge
XY	flux, charge-flux, phase, cavity
Heisenberg	spin, donor atom

couplings beyond nearest neighbors can be induced using a "quantum bus" (e.g., cavity) [J. Majer *et al.*, Nature (2007)]

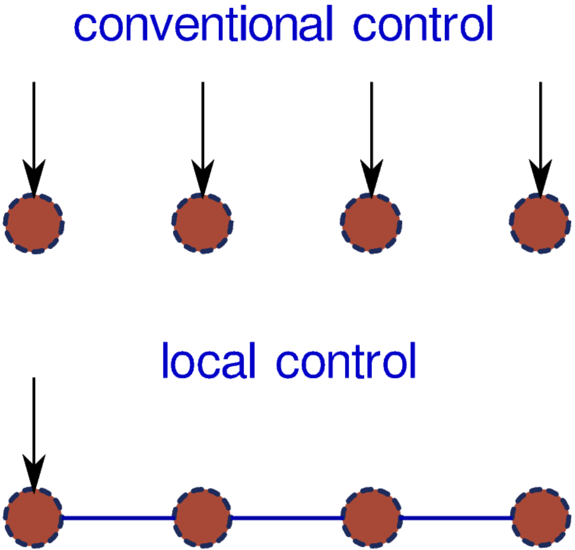
$$H(t) = H_0 + \sum_{j=1}^{p} f_j(t) H_j$$

$f_j(t)$ – control fields

- **State-selective control**: How to steer a quantum system from a given initial state to a pre-determined final state?
- **Operator (state-independent) control**: How to realize a desired unitary transformation (target quantum gate)?

The system is **completely controllable** if $H(t)$ can give rise to an arbitrary unitary transformation on its Hilbert space

Quantum control in qubit arrays with "always-on" interactions



$$H_0 = J\sum_{i=1}^{N-1}\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y + \Delta\sigma_i^z\sigma_{i+1}^z\right)$$

local control Hamiltonian: $H_c(t) = \underbrace{h_x(t)}_{f_1(t)}\underbrace{\sigma_1^x}_{H_1} + \underbrace{h_y(t)}_{f_2(t)}\underbrace{\sigma_1^y}_{H_2}$

suffices to show that the dimension of the dynamical Lie algebra $\mathcal{L}_{xy} \equiv \mathrm{span}\{-iH_0, -i\sigma_1^x, -i\sigma_1^y\}$ equals $d^2 - 1$ $(d \equiv 2^N)$

$\Rightarrow \mathcal{L}_{xy} \cong su(d) \Rightarrow e^{\mathcal{L}_{xy}} \cong SU(d)$ **(complete controllability)**

Control objectives (target gates)

CNOT on the last two qubits of the array:

$$\mathbf{CNOT}_N \equiv \underbrace{\mathbf{I}\otimes\ldots\otimes\mathbf{I}}_{N-2}\otimes\Big(\underbrace{|0\rangle\langle 0|\otimes\mathbf{I}+|1\rangle\langle 1|\otimes\boldsymbol{\sigma}^{\mathrm{x}}}_{\mathbf{CNOT}}\Big)$$

flip (**NOT**) of the last qubit in the array:

$$\mathbf{X}_N \equiv \underbrace{\mathbf{I}\otimes\ldots\otimes\mathbf{I}}_{N-1}\otimes\boldsymbol{\sigma}^{\mathrm{x}}$$

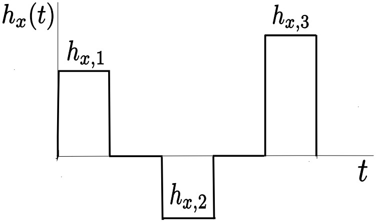
sufficient to use an x control field!

$\sqrt{\mathrm{SWAP}}$ on the last two qubits of the array:

$\sqrt{\mathrm{SWAP}_N}$ reminder: $\sqrt{\mathrm{SWAP}} \equiv e^{-i\frac{\pi}{8}}\, e^{i\frac{\pi}{8}(\boldsymbol{\sigma}^x\otimes\boldsymbol{\sigma}^x+\boldsymbol{\sigma}^y\otimes\boldsymbol{\sigma}^y+\boldsymbol{\sigma}^z\otimes\boldsymbol{\sigma}^z)}$

Control pulses and fidelity maximization

alternate x and y (or x only !) piecewise-constant controls:



full time evolution (total time $t_f \equiv N_t T$):

$$U(t_f) = U_{y,N_t/2} U_{x,N_t/2} \ldots U_{y,1} U_{x,1}$$

$$\left[U_{j,n} \equiv e^{-iH_{j,n}T} \;\; (j = x, y) \right]$$

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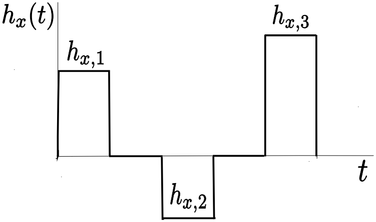

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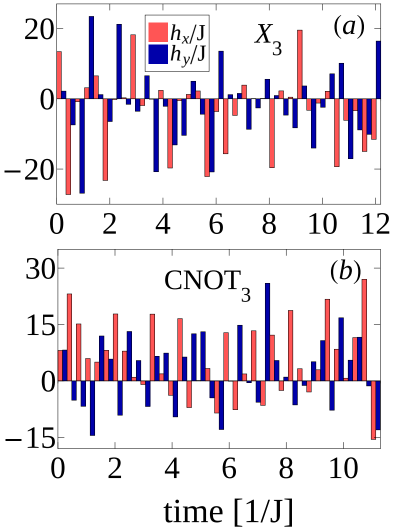
gate fidelity: $$F(t_f) = \frac{1}{d} \left| \mathrm{tr}\left[U^\dagger(t_f) U_{\text{target}} \right] \right|$$

$$\left[0 \leq F(t_f) \leq 1 \right]$$

maximize $F = F(\{h_{x,n}; h_{y,n}\})$ numerically for $N = 3, 4$

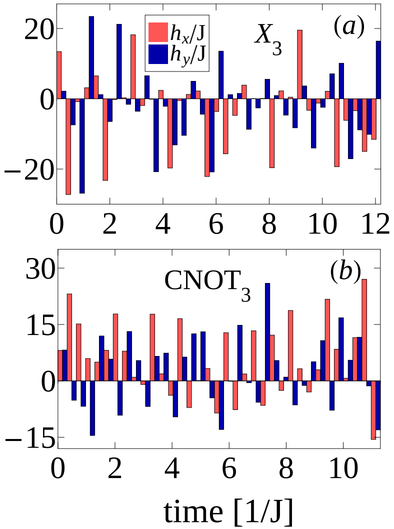


$\Delta \approx 5$ yields shortest times!

For $F \geq 1 - 10^{-3}$

$$t_{\mathrm{CNOT}_3} \approx 11.3\, J^{-1}$$

$$t_{\mathrm{CNOT}_4} \approx 4.5\, t_{\mathrm{CNOT}_3}$$



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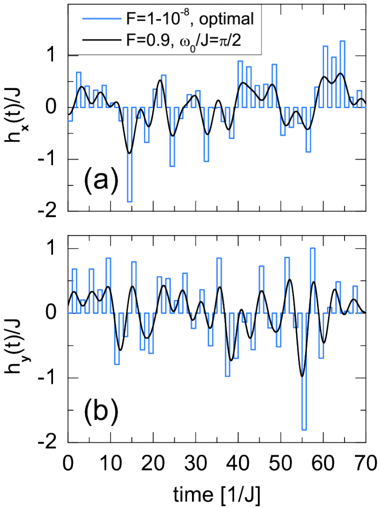
for $\boldsymbol{\Delta} \gtrsim \mathbf{11}$ shorter $\boldsymbol{X_3}$ gate times for $\boldsymbol{x}$-only control!

$\Rightarrow$ design principle for

superconducting charge qubits:

$$E_C/E_J \leftrightarrow \Delta$$



CNOT$_3$

practical limitation:

control fields cannot have arbitrarily high frequencies!

frequency-filtered control fields:

$$\tilde{h}_j(t) = \mathcal{F}^{-1}\{f(\omega)\mathcal{F}[h_j(t)]\}$$

ideal low-pass filter:

$$f(\omega) = \theta(\omega+\omega_0) - \theta(\omega-\omega_0)$$

ω_0 – cutoff frequency

Toffoli-gate realization with superconducting qubits

state of the art: two-qubit gates with $F > 90\%$ [DiCarlo *et al.* (2009)]

TOFFOLI $\equiv$ controlled-controlled-NOT

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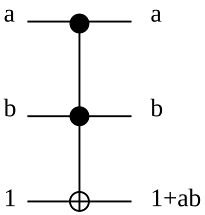

- trapped-ion [$F \approx 71\%$], photonic [$F \approx 81\%$] realizations in 2009!
- conventional **6 CNOTs + 10 single-qubit operations** approach not feasible due to long gate times!
- Way out: **use third level**
  A. Fedorov *et al.*, Nature (2012) : $F = 64.5 \pm 0.5\ \%$
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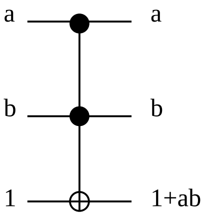

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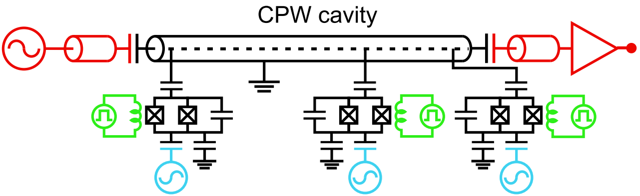
Can quantum control be of help?

$$F \xrightarrow{\text{decoherence}} F * \exp(-t_g/T_2)$$



effective XY-type model:

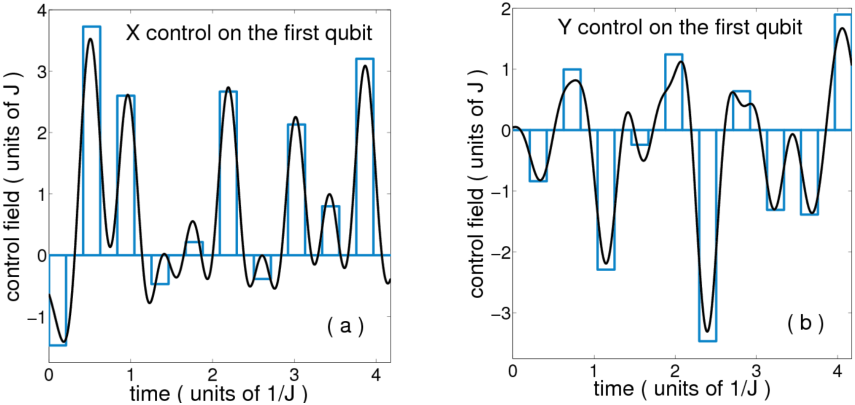
$$H_0 = \sum_{i<j} J_{ij}(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$$

$J_{12} = J_{23} = J \approx 30$ MHz, $J_{13} \approx 5$ MHz

$$H_c(t) = \sum_{i=1}^{3} \left[\Omega_x^{(i)}(t)\, \sigma_i^x + \Omega_y^{(i)}(t)\, \sigma_i^y\right]$$

$\sqrt{\Omega_x^2 + \Omega_y^2} \lesssim 130$ MHz

Toffoli gate in circuit QED: results



cutoff frequency:

$\omega_0 = 500\,\mathrm{MHz}$

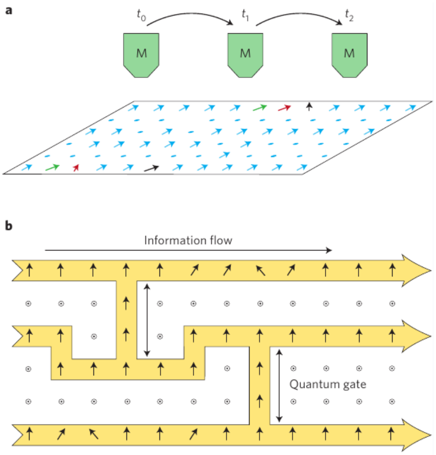
$\omega_0 \approx 17J$!

$t_g \approx 140\,\mathrm{ns}$ $\quad F \approx 99\%$

$t_g = 75\,\mathrm{ns}$ $\quad F \approx 92\%$

Measurement-based quantum computation (MQC)

R. Raussendorf and H. J. Briegel, PRL **86**, 5188 (2001)



H. J. Briegel *et al.*, Nature Phys. **5**, 19 (2009)

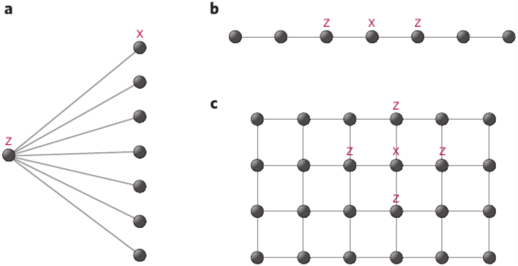
with local (single-qubit) measurements:
MQC ⟶ one-way QC

2D cluster state is a universal resource for MQC!

Other candidates, e.g., the AKLT state, are difficult to produce in solid-state systems!

one-way quantum computing

H. J. Briegel and R. Raussendorf, PRL **86**, 910 (2001)



initial preparation:

$$|G\rangle = \prod_{\{i,j\}} U_{\rm PG}^{(ij)} |+\rangle^{\otimes N}$$

$$|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

$$U_{\rm PG} = \mathrm{diag}(1,1,1,-1)$$

correlation operators $K_i \equiv \sigma_i^x \bigotimes_{j\in \mathrm{nghd}(i)} \sigma_j^z$ satisfy $K_i|G\rangle = \pm|G\rangle$

Stabilizer Hamiltonian

cluster states are eigenstates of the stabilizer Hamiltonian

$$H_{\text{stab}} = -\sum_i K_i$$

How to "generate" a stabilizer Hamiltonian starting from a "natural" two-body spin-1/2 (qubit) Hamiltonian?

$$H = H_0 + H_{\text{int}} \qquad H_0 = \sum_i (\Omega_i \sigma_i^x + \varepsilon_i \sigma_i^z)$$

Ising: $H_{\text{int}} = J \sum_i \sigma_i^z \sigma_{i+1}^z$

XY: $H_{\text{int}} = J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y)$

Heisenberg: $H_{\text{int}} = J \sum_i (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \sigma_i^z \sigma_{i+1}^z)$

Stabilizer Hamiltonian from Ising-type interactions

starting single-qubit Hamiltonian: $$H_s = \Omega(\sigma_1^y + \sum_{i=2}^{N-1} \sigma_i^x + \sigma_N^y)$$

What $e^{-i\theta\sum_i \sigma_i^z\sigma_{i+1}^z} H_s\, e^{i\theta\sum_i \sigma_i^z\sigma_{i+1}^z}$ amounts to?

basic relations:

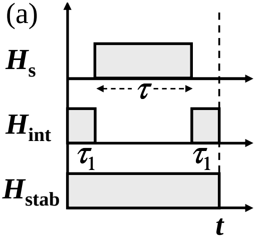
$$e^{-i\theta\sigma_1^z\sigma_2^z}\sigma_1^{x,y}e^{i\theta\sigma_1^z\sigma_2^z} = \cos(2\theta)\sigma_1^{x,y} \pm \sin(2\theta)\sigma_1^{y,x}\sigma_2^z$$

special case $\theta = \pi/4$ – increasing the order of the Pauli-matrix terms:

$$e^{-i\frac{\pi}{4}\sigma_1^z\sigma_2^z}\sigma_1^x e^{i\frac{\pi}{4}\sigma_1^z\sigma_2^z} = \sigma_1^y\sigma_2^z \quad ; \quad e^{-i\frac{\pi}{4}\sigma_1^z\sigma_2^z}\sigma_1^y e^{i\frac{\pi}{4}\sigma_1^z\sigma_2^z} = -\sigma_1^x\sigma_2^z$$

$\Rightarrow$ 1D stabilizer Hamiltonian; 1D $\rightarrow$ 2D straightforward!

Stabilizer Hamiltonian as the effective Hamiltonian



Ising-interaction pulses with $\tau_1 \equiv \pi/(4J)$:

$$H_{\text{stab}} = e^{-i\frac{\pi}{4}\sum_i \sigma_i^z\sigma_{i+1}^z} H_{\text{s}}\, e^{i\frac{\pi}{4}\sum_i \sigma_i^z\sigma_{i+1}^z}$$

state evolution:

$$\rho(0) \xrightarrow{\tau_1 H_{\text{int}}} \xrightarrow{\tau H_{\text{s}}} \xrightarrow{-\tau_1 H_{\text{int}}} \rho(t = \tau + 2\tau_1)$$

pulse-induced effective evolution:

$$e^{-i\tau H_{\text{stab}}} = \exp\left(-i\frac{\pi}{4}\sum_i \sigma_i^z\sigma_{i+1}^z\right) e^{-i\tau H_{\text{s}}} \exp\left(i\frac{\pi}{4}\sum_i \sigma_i^z\sigma_{i+1}^z\right)$$

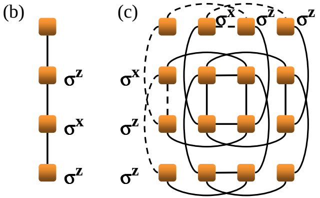
Note: the ability to switch the interactions on/off is required!

[e.g., G. Wendin and V. S. Shumeiko (2005)]

Stabilizer Hamiltonian from XY-type interactions

main difference from the Ising case: $e^{-i\theta\sum_i(\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y)}$ does not factorize as $[\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y,\sigma_{i+1}^x\sigma_{i+2}^x+\sigma_{i+1}^y\sigma_{i+2}^y]\neq 0 \;\Rightarrow\; H_{\rm stab}^{\rm 2D}$ step by step:

1. generate $H_{\rm stab}$ for adjacent qubit pairs
2. connect the pairs to obtain $H_{\rm stab}^{\rm 1D}$
3. generate multiple $H_{\rm stab}^{\rm 1D}$ and connect them pairwise into ladders
4. connect the ladders to obtain $H_{\rm stab}^{\rm 2D}$



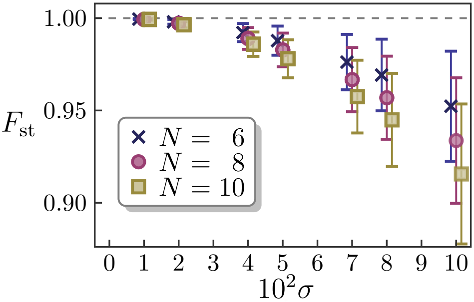
the obtained $H_{\rm stab}$ is **twisted**!

the corresponding cluster state is twisted too!

Cluster-state fidelity: numerical results in the XY case

$$F_{\rm st}(\tau) \equiv |\langle \Psi_{00\ldots0}|U_\tau(\delta)|\Psi_{00\ldots0}\rangle|^2$$

$$U_\tau(\delta) \equiv e^{-i\tau H_{\rm stab}(\pi/4+\delta)}$$



analytical (perturbative) result:

$$1 - F_{\rm st} \propto \delta^2 \quad (\,\delta \ll \pi/4\,)$$

$$\delta \;\rightarrow\; \delta_i \quad (i = 1, \ldots, N-1)$$

averaged over $\mathbf{2000}$ random realizations of the δ_i taken from a Gaussian distribution of width σ

Conclusions and Outlook

- Local-control approach allows for efficient realization of quantum gates in qubit arrays with XXZ Heisenberg interaction!
- Using quantum control, within only $\mathbf{75}$ ns a Toffoli gate is predicted to be realized with intrinsic fidelity above $\mathbf{92\%}$!
- 2D cluster states, a universal resource for MQC, can be preserved with high fidelity in XY- and Ising-coupled qubit arrays!

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