Spin-Hall conductivity due to Rashba spin-orbit interaction in disordered systems

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go to conclusions

Outline

- Spin-Hall effect in pure material
- Kubo-Greenwood formula
- Zero-loop approximation
- Weak localization correction
- Spin current and the magnetization
- Conclusions

Spin-Hall effect

J. Sinova et al. 2004: The case of no impurities

$$\sigma^z_{yx} = \frac{e}{8\pi}$$

J. Inoue et al., 2004:

In the diffusive case, due to the vertex renormalization,

$$\sigma^z_{yx} = 0$$

Other papers confirmed this result:

Mishchenko et al., Khaetskii,

Raimondi & Schwab, Dimitrova.

Our scope: improve the precision of the calculation.

General relations

The Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \alpha(\hat{\sigma}^1 \hat{p}_y - \hat{\sigma}^2 \hat{p}_x) + U(\vec{r}),$$

where $U(\vec{r})$ is a disorder potential,
$\langle U(\vec{r})U(\vec{r}\,')\rangle = \frac{1}{2\pi\nu\tau}\delta(\vec{r} - \vec{r}\,')$.
$\alpha(\hat{\sigma}^1 \hat{p}_y - \hat{\sigma}^2 \hat{p}_x)$ is a Rashba SO term,
which leads to a modification of the charge current operator:

$$\hat{\vec{p}} \rightarrow \hat{\vec{p}} - \frac{e}{c}\vec{\tilde{A}},$$

$\vec{\tilde{A}} = -(\alpha mc/e)(-\hat{\sigma}^2, \hat{\sigma}^1, 0) =$ effective vector potential.

General relations

The Hamiltonian:

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Evaluate spin current assuming:

- Linear responce in electric field $\Longrightarrow$ Kubo formula.
- We use the disorder averaging diagrammatic technique, $p_{\mathrm{F}} l \gg 1$.
- Rashba SOI $\alpha p_{\mathrm{F}} \ll E_{\mathrm{F}}$

A generalized Kubo-Greenwood formula

$$\sigma_{yx}^{z} = \frac{e}{2\pi m^2} \overline{\mathrm{Tr}\left[\frac{\sigma^3}{2} p_y \hat{G}_{\mathrm{R}} \left(p_x - \frac{e}{c}\tilde{A}_x\right) \hat{G}_{\mathrm{A}}\right]}$$

Loop expansion (click here for more information):

$$\sigma_{yx}^{z} = |e| \sum_{n \geq 0} \frac{s_n}{(p_{\mathrm{F}} l)^n}, \qquad p_{\mathrm{F}} l \gg 1,$$

n =number of loops, l =mean scattering free path

The charge current operator

$$\hat{p}_x - \frac{e}{c}\tilde{A}_x = p_{\mathrm{F}} \hat{n}_x + \left(\hat{p}_x - p_{\mathrm{F}} \hat{n}_x - \frac{e}{c}\tilde{A}_x\right), \qquad \hat{n}_x \equiv \frac{\hat{p}_x}{p}$$

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- In zero loop diagrams, gives the contribution $\propto e(p_\mathrm{F} l)$ to σ^z_{yx}
- In first loop diagrams, gives the contribution $\propto e$ to σ^z_{yx}

A generalized Kubo-Greenwood formula

$$\sigma^z_{yx} = \frac{e}{2\pi m^2}\overline{\mathrm{Tr}\left[\frac{\sigma^3}{2} p_y \hat{G}_{\mathrm{R}}\left(p_x - \frac{e}{c}\tilde{A}_x\right)\hat{G}_{\mathrm{A}}\right]}$$

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- In zero loop diagrams, gives the contribution $\propto e$ to σ^z_{yx}
- In first loop diagrams, gives the contribution $\propto \frac{e}{p_{\mathrm{F}} l}$ to σ^z_{yx}

Thus,

taking into account the contribution from

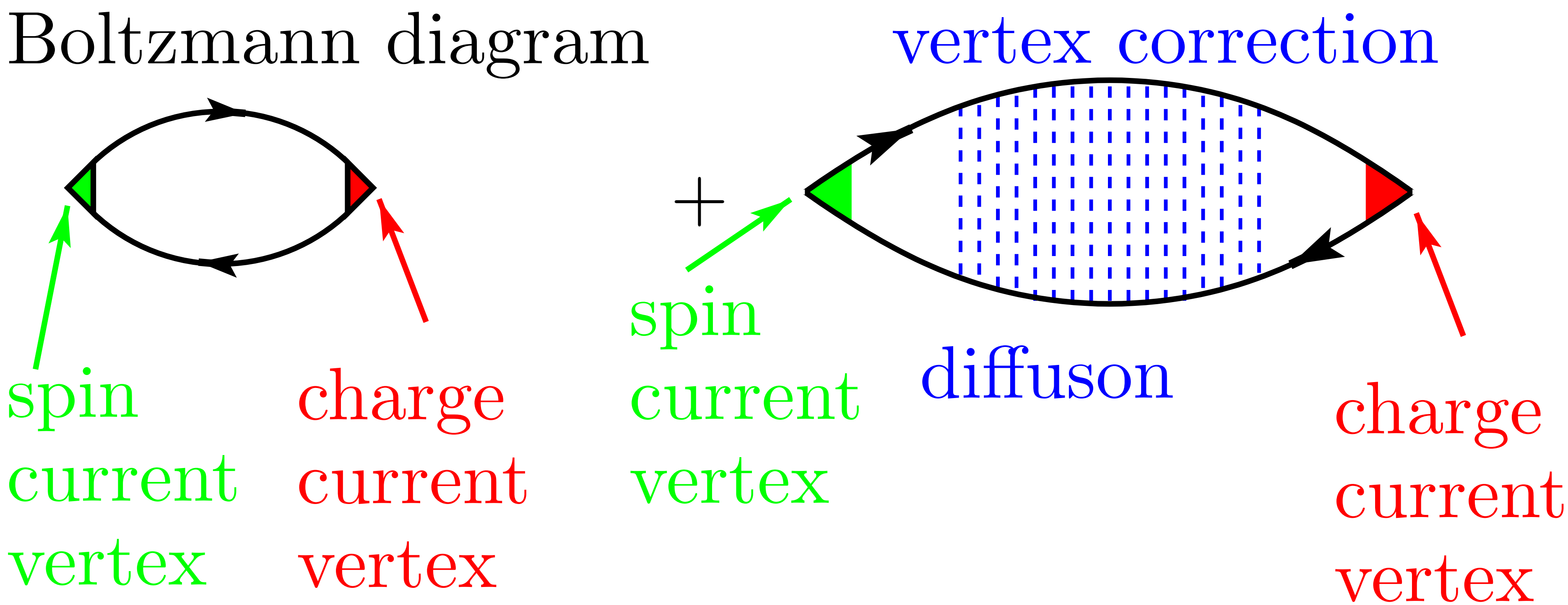
$$\left(\hat{p}_x - p_{\mathrm{F}}\hat{n}_x - \frac{e}{c}\tilde{A}_x\right)$$

in zero-loop diagrams means we have to consider the contribution from

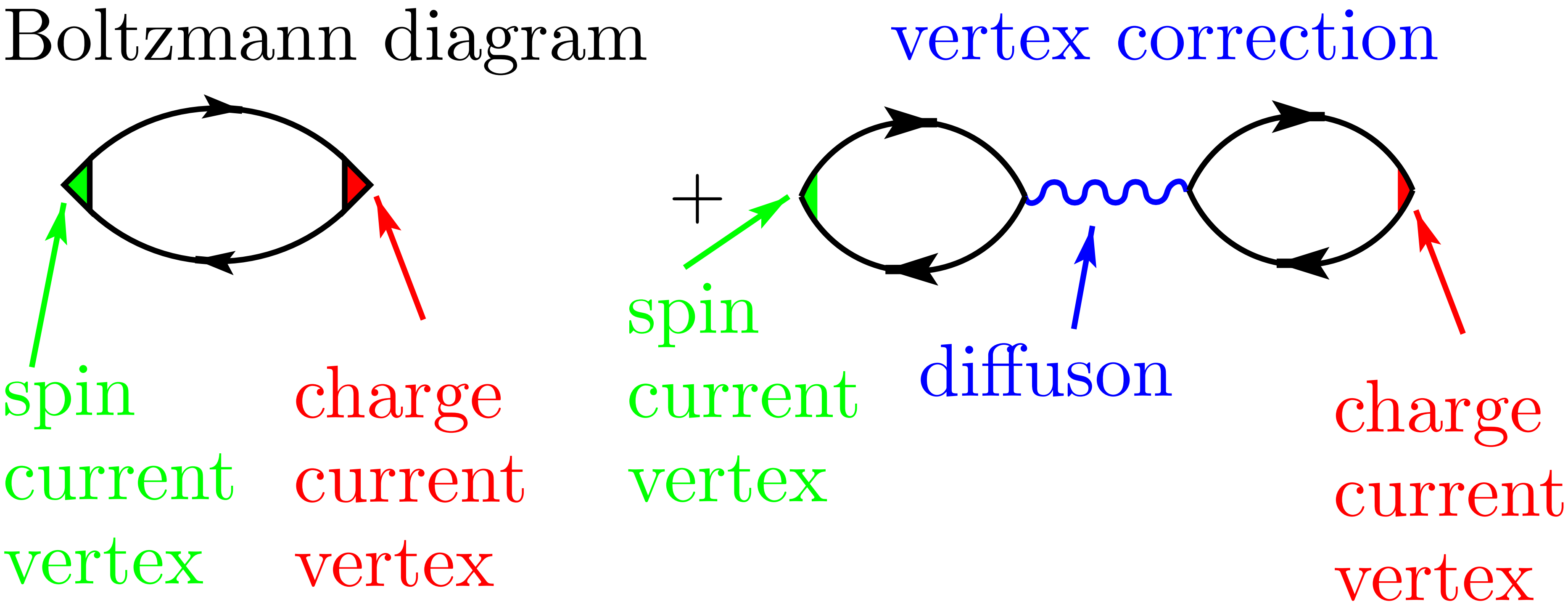
$$p_{\mathrm{F}}\hat{n}_x$$

in first-loop diagrams.

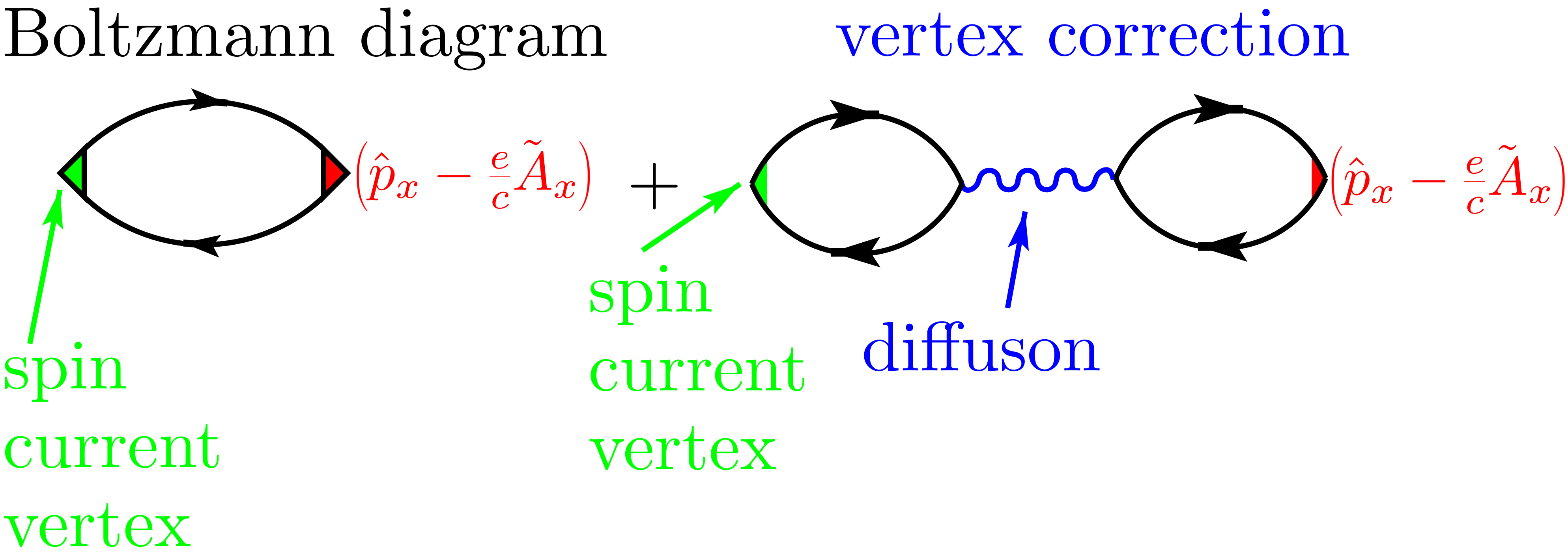
Zero-loop approximation



Zero-loop approximation

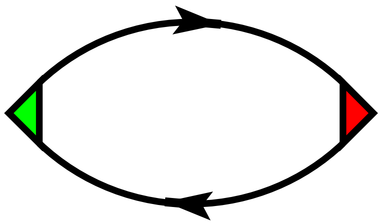
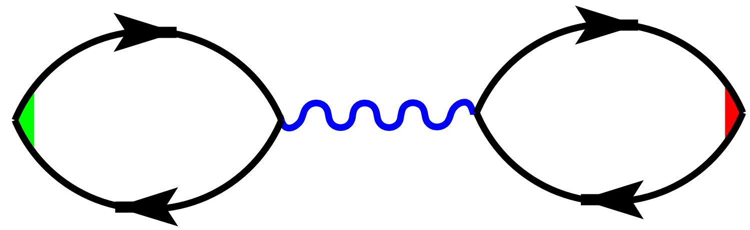


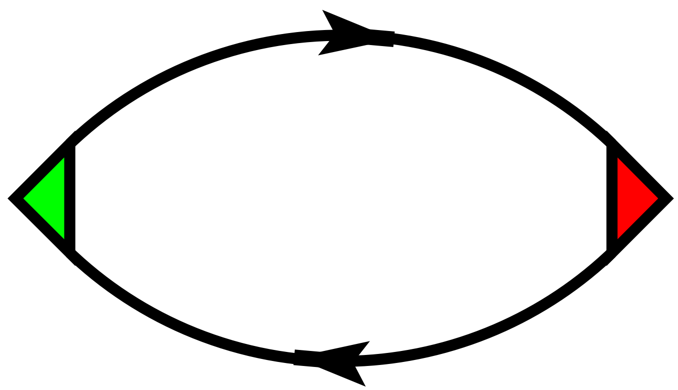
Zero-loop approximation



Zero-loop approximation

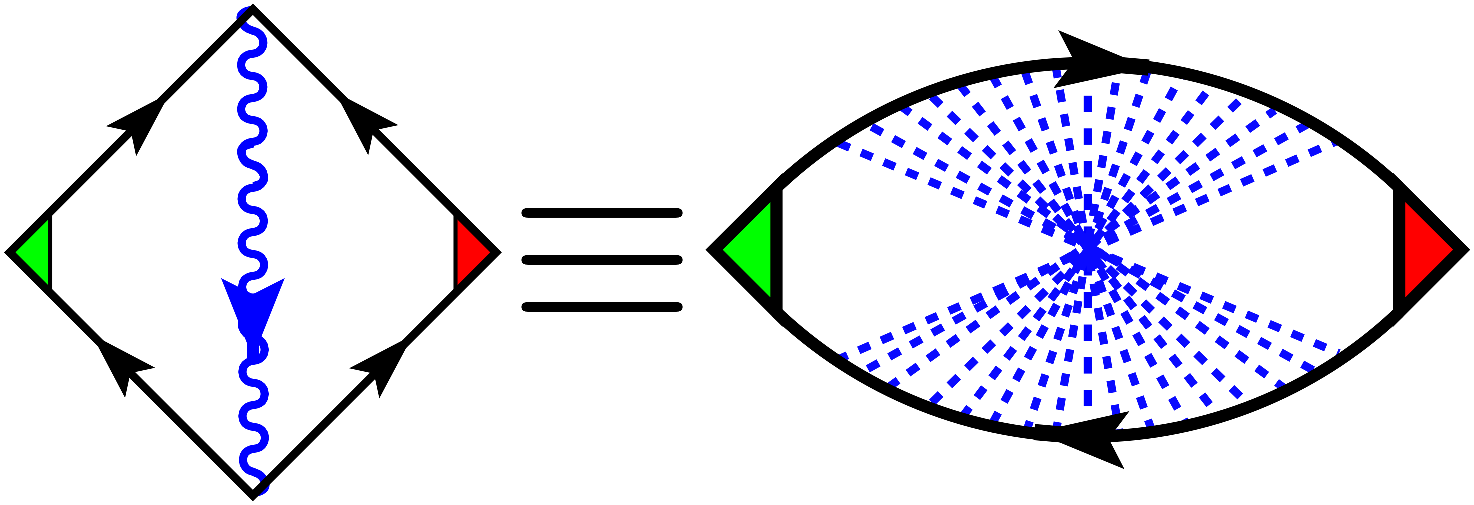
Boltzmann diagram vertex correction


$\left(\hat{p}_x - \frac{e}{c}\tilde{A}_x\right)$ $+$

$\left(\hat{p}_x - \frac{e}{c}\tilde{A}_x\right)$

$=$

$\hat{p}_x = 0$

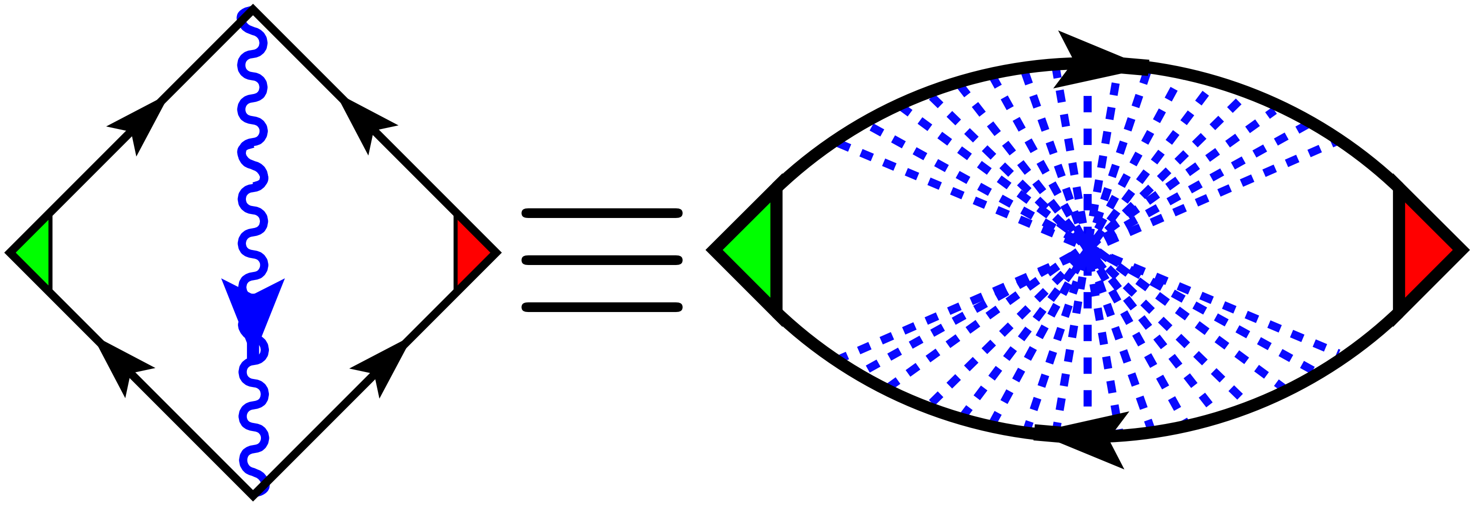
The renormalization of the charge current vertex results in the cancellation of the anomalous term $\frac{e}{c}\vec{\tilde{A}}$ in the current vertex ($\omega = 0$)

Weak localization correction



about Cooperon and diffuson...

Weak localization correction

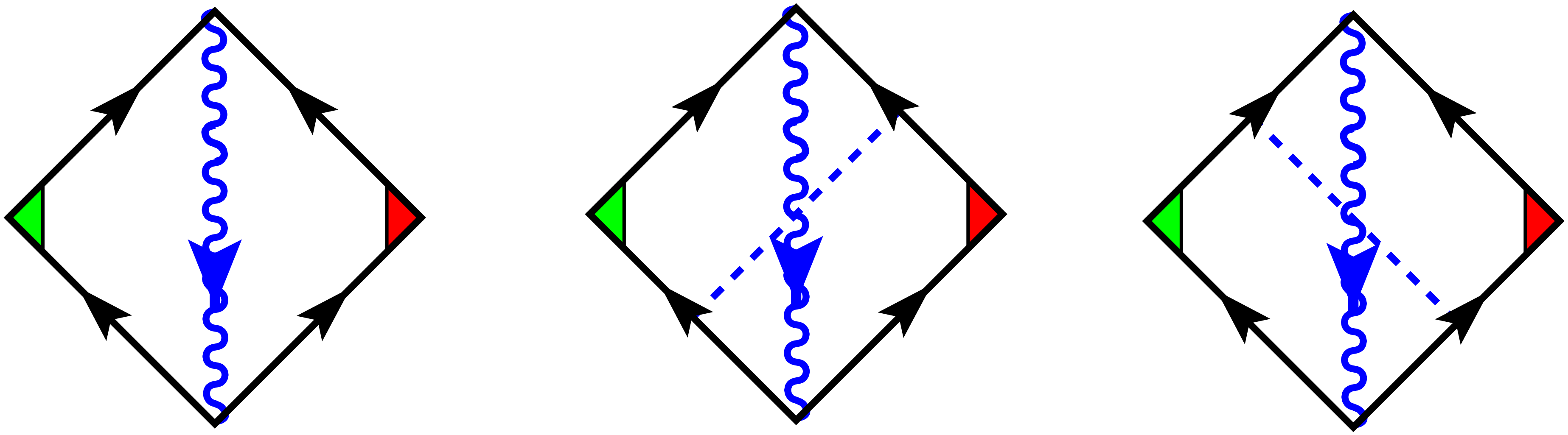


$$= \frac{e}{2\pi m^2} \sum_{\gamma\gamma'=0}^{3} \mathrm{Tr} \left\{ \int \frac{\mathrm{d}^2 p}{(2\pi)^2} G_\mathrm{A}(\vec{p}) p_y \frac{\sigma^3}{2} G_\mathrm{R}(\vec{p}) \sigma^\gamma \times \right.$$

$$\left. \left[\sigma^{\gamma'} G_\mathrm{R}(\vec{q} - \vec{p}) \left(-p_x - \frac{e}{c} \tilde{A}_x \right) G_\mathrm{A}(-\vec{p}) \right]^T \right\} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} C^{\gamma\gamma'}(\vec{q})$$

about Cooperon and diffuson...

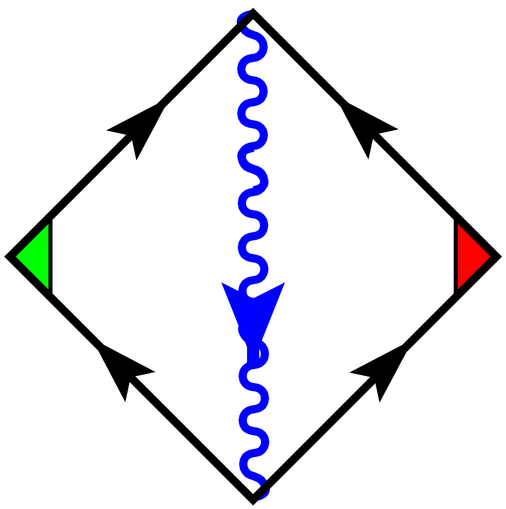
Weak localization correction



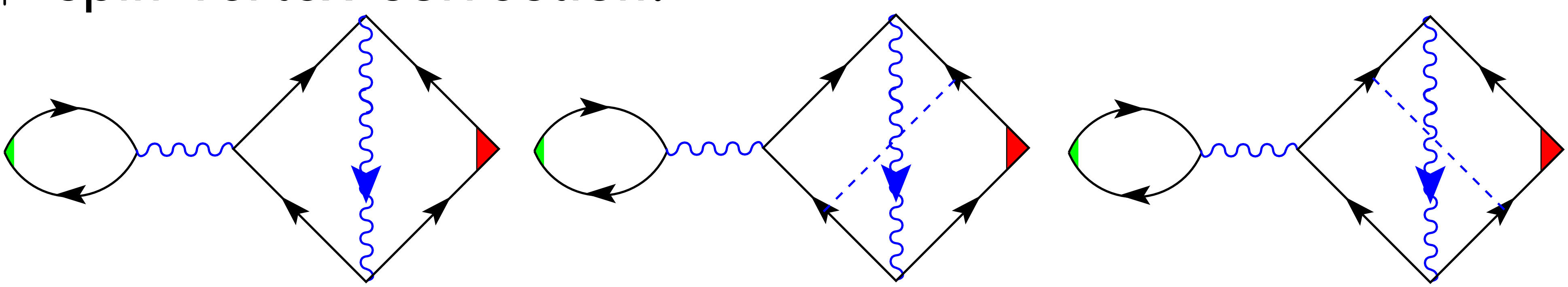
Click here for the expression.

about Cooperon and diffuson...

Weak localization correction



+ spin vertex correction:

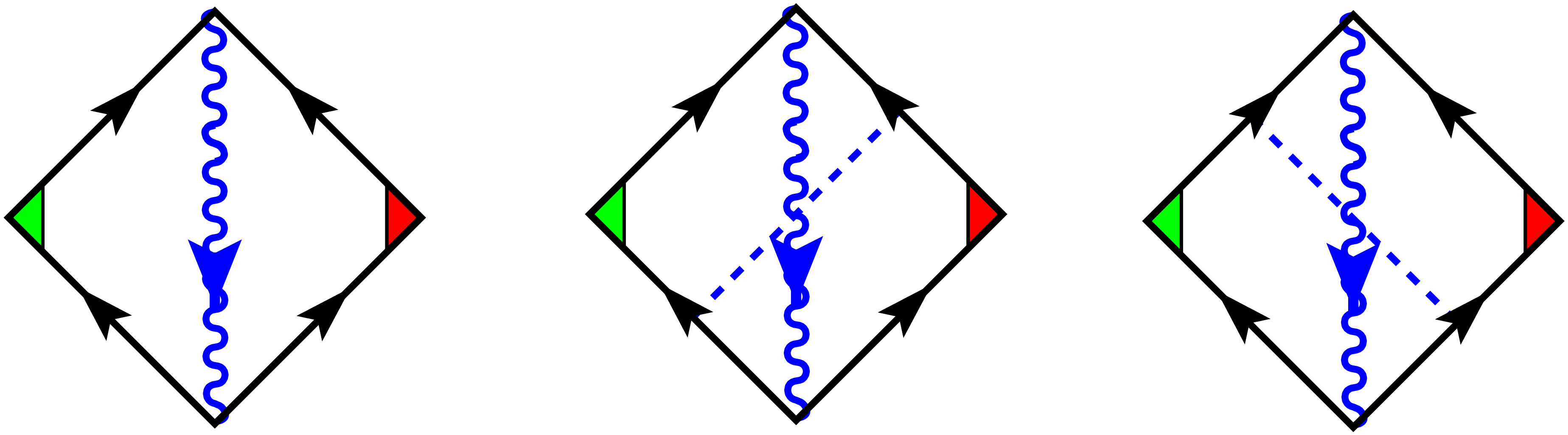


Note:

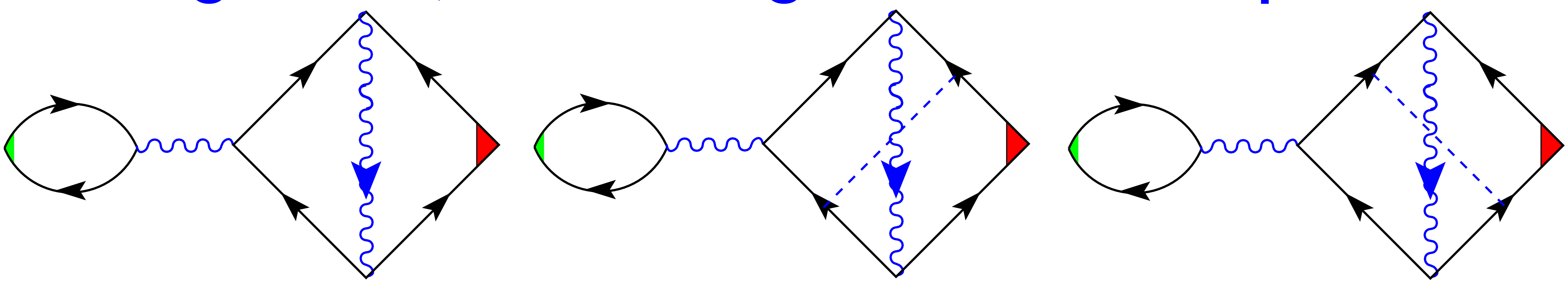
the charge-current vertex renormalization is irrelevant here

about Cooperon and diffuson...

Weak localization correction



All together, the diagrams add up to zero



about Cooperon and diffuson...

Connection between the spin current and the magnetization

Without magnetic field and for non-magnetic impurities

$$\dot{\hat{s}}_k(t) = -2m\alpha \dot{\hat{j}}_k^{s_z}(t), \quad k = x, y$$

- valid for arbitrary $\vec{E}$ and for systems with interaction.

The magnetization

$$\langle \dot{\hat{s}}_k \rangle(t) = -2m\alpha \langle \dot{\hat{j}}_k^{s_z} \rangle(t)$$

In diffusive systems, stationary state is reached at $t \to \infty$.
$\Longrightarrow$ the spin-Hall current must be zero at $\omega = 0$.

Thus,

the result of J. Sinova et al. 2004

for a clean sample

$$\sigma^{z}_{yx} = \frac{e}{8\pi}$$

means that

$$\langle \hat{s} \rangle_y \longrightarrow \infty, \quad t \longrightarrow \infty.$$

Generalization for the Dresselhaus term

$$\hat{H}'(\hat{\vec{p}}, \vec{r}) = \frac{\hat{p}^2}{2m} + U(\vec{r}) + \alpha\left(\hat{\sigma}^1\hat{p}_y - \hat{\sigma}^2\hat{p}_x\right) + \\ +\beta\left(\hat{\sigma}^1\hat{p}_x - \hat{\sigma}^2\hat{p}_y\right) + e\vec{r}\vec{E},$$

$$-\frac{1}{2m}\dot{\hat{s}}_x(t) = \alpha\hat{j}_x^{s_z}(t) + \beta\hat{j}_y^{s_z}(t),$$
$$-\frac{1}{2m}\dot{\hat{s}}_y(t) = \alpha\hat{j}_y^{s_z}(t) + \beta\hat{j}_x^{s_z}(t),$$
$$\hat{j}_x = -\frac{1}{2m}\frac{\alpha\dot{\hat{s}}_x - \beta\dot{\hat{s}}_y}{\alpha^2 - \beta^2}, \quad \hat{j}_y = -\frac{1}{2m}\frac{\beta\dot{\hat{s}}_x - \alpha\dot{\hat{s}}_y}{\beta^2 - \alpha^2}, \quad \alpha \neq \beta.$$

$$\lim_{t\to\infty}\langle\hat{\vec{j}}^{s_z}\rangle(t) = 0 \Longleftrightarrow \lim_{t\to\infty}\langle\hat{s}_k\rangle(t) = \text{const.}, \quad k = x, y.$$

Click me for the case $\alpha = \beta$...

Conclusions

- We have calculated the zero-loop and the weak localization contributions to the σ^z_{yx}.
- Both contributions result zero independently.
- General argument: spin-Hall current is zero at $\omega = 0$ and for $B = 0$.

Thanks to: Evgenii Mishchenko & Andrei Shytov.

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this document is available on http://shalaev.pochta.ru and here.

References

disorder averaging diagrammatic technique:

A. A. Abrikosov, L. P.Gor'kov and I. E. Dzyaloshinskii, Methods of quantum field theory in statistical physics, Dobrosvet (Moscow), 1998.

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Diffuson and Cooperon

$$X_D^{\alpha\beta}(\vec{q}) = \frac{1}{4\pi\nu\tau}\int\frac{\mathrm{d}^2p}{(2\pi)^2}\mathrm{Tr}[\sigma^\alpha G_\mathrm{R}(\vec{p})\sigma^\beta G_\mathrm{A}(\vec{p}-\vec{q})],$$

$$X_C^{\alpha\beta}(\vec{q}) = \frac{1}{4\pi\nu\tau}\int\frac{\mathrm{d}^2p}{(2\pi)^2}\mathrm{Tr}[\sigma^\alpha G_\mathrm{R}(\vec{p})\sigma^\beta G_\mathrm{A}^T(\vec{q}-\vec{p})],$$

$$D^{\alpha\alpha} = \frac{1}{4\pi\nu\tau}\frac{1}{1-X_D^{\alpha\alpha}}, \quad C^{\alpha\alpha'}(\vec{q}) = \frac{1}{4\pi\nu\tau}\left[\frac{X_\mathrm{C}(\vec{q})}{1-X_\mathrm{C}(\vec{q})}\right]_{\alpha\alpha'}$$

See my unofficial notes for more information.

Back

The case of $\alpha = \beta$

$$\hat{H}_{\rm R}(\hat{\vec{p}}', \vec{r}\,') \equiv \hat{H}'(R_{\pi/4}\hat{\vec{p}}, R_{\pi/4}\vec{r}\,) =$$

$$= \frac{\hat{p}'^{2}}{2m} - 2\alpha\hat{\sigma}^{2'}\hat{p}'_x + U'(\vec{r}\,'\,) + e\vec{r}\,'\vec{E}' \equiv \hat{H}_{\rm R0} + e\vec{r}\,'\vec{E}',$$

$$\hat{\sigma}^{12'} = \frac{1}{\sqrt{2}}(\hat{\sigma}^2 \pm \hat{\sigma}^1), \qquad \hat{\sigma}'_3 \equiv \hat{\sigma}^3.$$

$$\hat{\rho}(t<0) = \hat{\rho}_0 = e^{-\hat{H}_{\rm R0}/T}/Z, \qquad \langle \hat{\vec{j}}\,'^{s_z}\rangle(t=0) = 0.$$

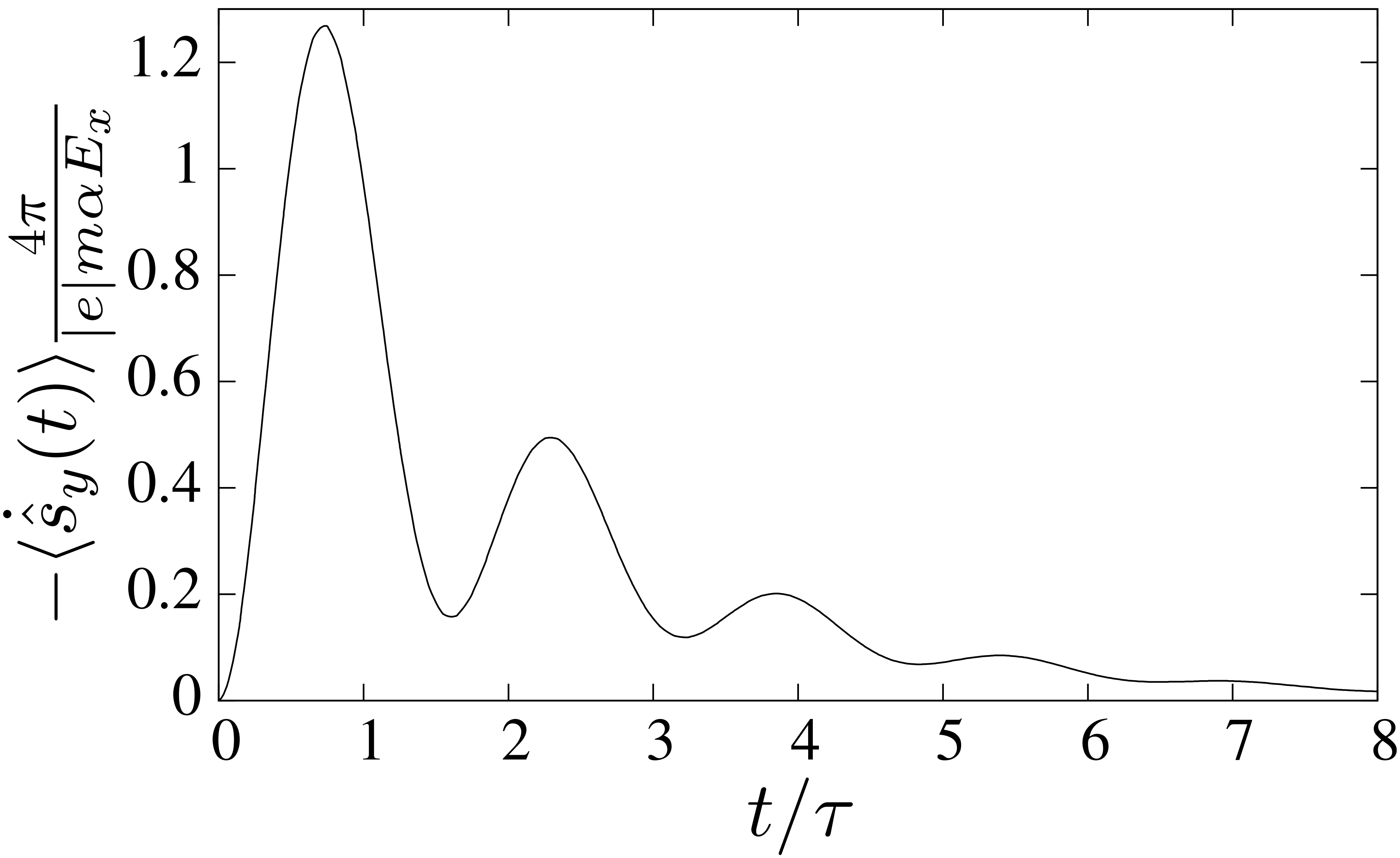
$$\langle \hat{\vec{j}}\,'^{s_z}\rangle(t) = {\rm Tr}\left[\frac{\hat{\sigma}^3}{2}\frac{\hat{\vec{p}}}{m}e^{-i\hat{H}_{\rm R}t}\hat{\rho}_0 e^{i\hat{H}_{\rm R}t}\right].$$

$$\mathop{\rm Tr}_{\rm spin}[\hat{\sigma}^{k'}\hat{H}_{\rm R}] = 0, \qquad \mathop{\rm Tr}_{\rm spin}[\hat{\sigma}^{k'}\hat{H}_{\rm R0}] = 0, \qquad k = 1, 3,$$

$$\Longrightarrow \mathop{\rm Tr}_{\rm spin}\left[\frac{\hat{\sigma}^3}{2}\frac{\hat{\vec{p}}}{m}e^{-i\hat{H}_{\rm R}t}\hat{\rho}_0 e^{i\hat{H}_{\rm R}t}\right] = 0, \qquad \forall t.$$

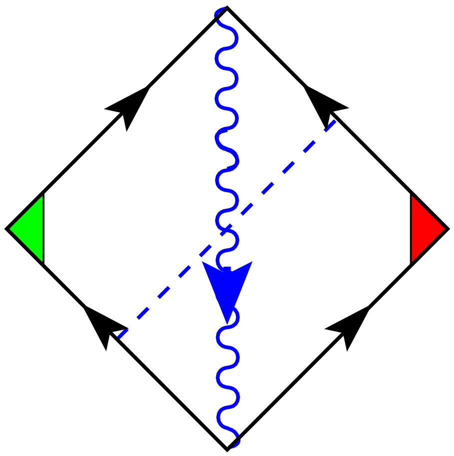
Spin-Hall current decay in time

(only the contribution from zero-loop diagrams)



The period of oscillation is $1/\Delta$, and the exponential decay time is τ.

Expression for one of the weak localization diagrams



$$= \frac{e}{2\pi m^2} \sum_{\gamma=0}^{3} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} C^{\gamma\gamma}(\vec{q}) \times \frac{1}{2m\tau} \sum_{\mu=0}^{3} A^{\gamma\mu} B^{\mu\gamma},$$

$$A^{\gamma\mu} = \mathrm{Tr}\left\{ \int \frac{\mathrm{d}^2 p}{(2\pi)^2} G^{<}(\vec{p})\sigma^{\gamma} G_{\mathrm{A}}^{T}(-\vec{p})\sigma^{\mu} \right\},$$

$$B^{\mu\gamma} = \mathrm{Tr}\left\{ \int \frac{\mathrm{d}^2 p'}{(2\pi)^2} \sigma^{\gamma} G^{>}(-\vec{p}\,') \left[G_{\mathrm{A}}(\vec{p}\,')\sigma^{\mu} \right]^{T} \right\},$$

$$G^{<}(\vec{p}) = G_{\mathrm{A}}(\vec{p}) p_y \frac{\sigma^3}{2} G_{\mathrm{R}}(\vec{p}),$$

$$G^{>}(-\vec{p}) = G_{\mathrm{R}}(-\vec{p}) \left(-p_x - \frac{e}{c}\tilde{A}_x \right) G_{\mathrm{A}}(-\vec{p}).$$

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