Spin-Hall conductivity due to Rashba spin-orbit interaction in disordered systems

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1 Introduction

The spin-Hall conductivity in the ballistic case [1, 2]

$$\sigma^z_{yx} = \frac{e}{8\pi}.$$

In the diffusive case, due to the vertex renormalization, [3]

$$\sigma^z_{yx} = 0.$$

Other papers confirmed this result, see [4, 5, 6].
Motivation: check the consistency of the calculation and improve its precision.

2 Hamiltonian and its eigensystem

Rashba spin-orbit interaction term modifies the Hamiltonian of a disordered system as follows:

$$\hat{H}' = \frac{\hat{p}^2}{2m} + \alpha(\hat{\sigma}^1\hat{p}_y - \hat{\sigma}^2\hat{p}_x) + U(\vec{r}), \tag{1}$$

where $\hat{\sigma}^{12}$ are Pauli matrices, and $U(\vec{r})$ is a short-range disorder potential: $\langle U(\vec{r})U(\vec{r}')\rangle = (2\pi\nu\tau)^{-1}\delta(\vec{r}-\vec{r}')$.
The spin-orbit term in the Hamiltonian (1) modifies expression for the current operator:

$$\hat{\vec{j}} = \frac{ie\hbar}{2m}\left[\left(\vec{\nabla}\hat{\psi}^\dagger\right)\hat{\psi} - \hat{\psi}^\dagger\vec{\nabla}\hat{\psi}\right] - \frac{e^2}{mc}\hat{\psi}^\dagger\left(\vec{A} + \vec{\tilde{A}}\right)\hat{\psi},$$

where together with an ordinary vector potential $\vec{A}$ a fictitious vector potential $\vec{\tilde{A}}$ is introduced:

$$\vec{\tilde{A}} = -\alpha mc/e \times (-\hat{\sigma}^2, \hat{\sigma}^1, 0)^T. \tag{2}$$

The spin-current operator is independent on $\vec{A}$:

$$\hat{\vec{j}}^{s_z} = -\frac{i\hbar^2}{4m}\left[\hat{\psi}^\dagger\hat{\sigma}^3\vec{\nabla}\hat{\psi} - (\vec{\nabla}\hat{\psi}^\dagger)\hat{\sigma}^3\hat{\psi} - \frac{2ie}{\hbar c}\vec{A}\hat{\psi}^\dagger(\vec{r})\hat{\sigma}^3\hat{\psi}(\vec{r})\right].$$

Due to the presence of a Pauli matrix in the expression for the current vertex, the diamagnetic term equals zero for the spin current.

3 A generalized Kubo-Greenwood formula

In Keldysh technique the spin current equals to

$$\overline{\vec{j}^{s_z}(\omega)} = -\frac{i\hbar^2}{2m}\int_{-\infty}^{\infty}\frac{dE}{2\pi}\overline{\mathrm{Tr}\left\{\left(\vec{p} - \frac{e}{\hbar c}\vec{A}\right)\frac{\hat{\sigma}^3}{2}\hat{G}_{\mathrm{K}}(E, E-\omega)\right\}}, \tag{3}$$

where G_{K} is the Keldysh component of the matrix 2x2 Green function, Tr stands for the trace in both momentum and spin spaces, and line denotes averaging over impurities configurations. Loop expansion:

$$\sigma^z_{yx} = |e|\sum_n \frac{s_n}{(p_{\mathrm{F}}l)^n}, \quad p_{\mathrm{F}}l \gg 1,$$

l =mean free path

The charge current operator

$$\hat{p}_x - \frac{e}{c}\tilde{A}_x = p_{\mathrm{F}}\hat{n}_x + \left(\hat{p}_x - p_{\mathrm{F}}\hat{n}_x - \frac{e}{c}\tilde{A}_x\right), \quad \hat{n}_x \equiv \frac{\hat{p}_x}{p}$$

The leading term term

- In zero loop diagrams, gives the contribution$\propto e(p_{\mathrm{F}}l)$ to σ^z_{yx}
- In first loop diagrams, gives the contribution$\propto e$ to σ^z_{yx}

The correction

- In zero loop diagrams, gives the contribution$\propto e$ to σ^z_{yx}
- In first loop diagrams, gives the contribution$\propto \frac{e}{p_{\mathrm{F}}l}$ to σ^z_{yx}

Thus, taking into account the contribution from $\left(\hat{p}_x - p_{\mathrm{F}}\hat{n}_x - \frac{e}{c}\tilde{A}_x\right)$ in zero-loop diagrams means we have to consider the contribution from $p_{\mathrm{F}}\hat{n}_x$ in first-loop diagrams.

4 Zero-loop approximation.

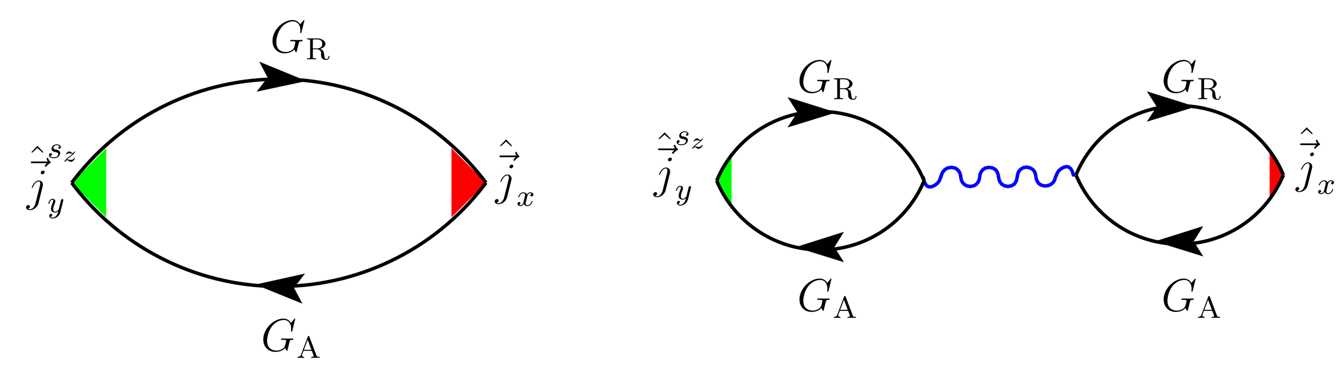
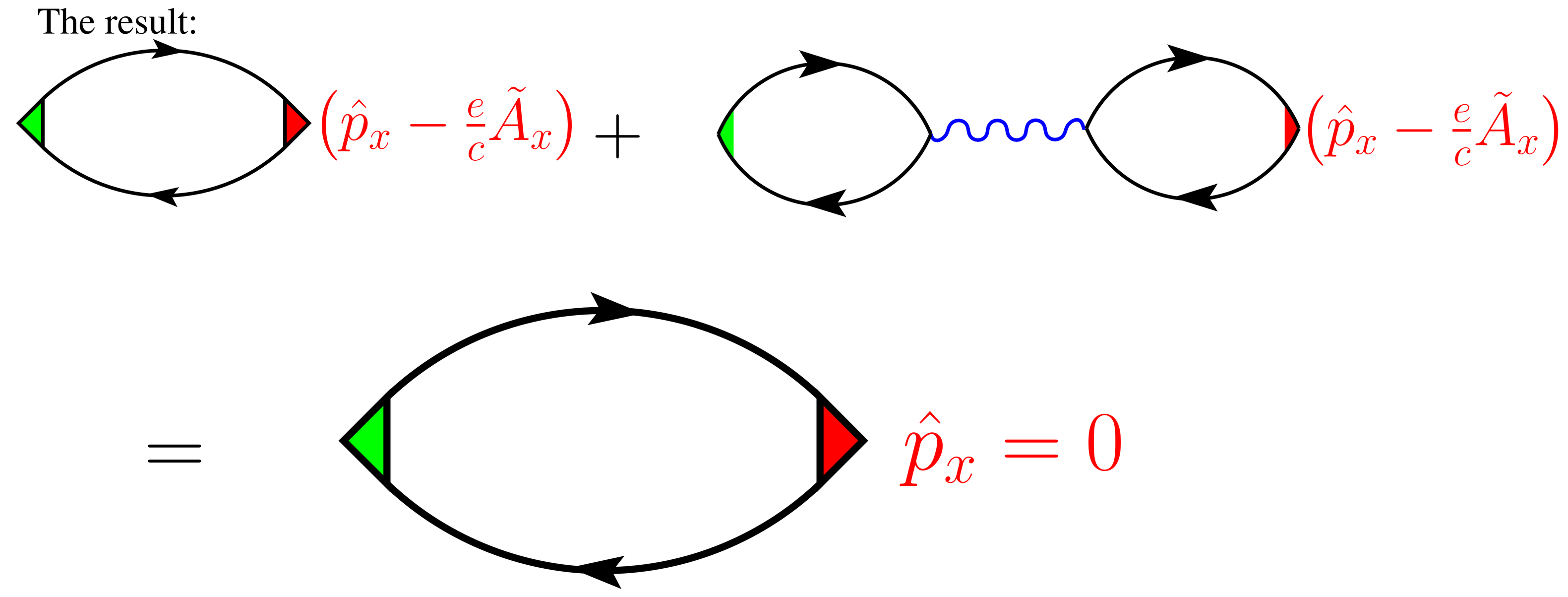


Figure 1. Diagrams for the spin Hall conductivity in the zero-loop approximation.
Wavy line denotes diffuson.

$$\sigma^z_{yx} = \frac{e}{2\pi m^2}\mathrm{Tr}\left[\frac{\sigma_3}{2}p_y\hat{G}_{\mathrm{R}}\left(p_x - \frac{e}{c}\hat{A}_x\right)\hat{G}_{\mathrm{A}}\right]. \tag{4}$$

One can generate diagrams of the disorder averaging technique from (4) by "dressing" it with diffuson and cooperon lines. In the zeroth loop approximation, the averaging of (4) produces the two diagrams in fig. 2.
The result:



$\Longrightarrow$ current vertex renormalization results in the cancellation of the anomalous term in the charge current vertex.

5 Weak localization correction (one loop)

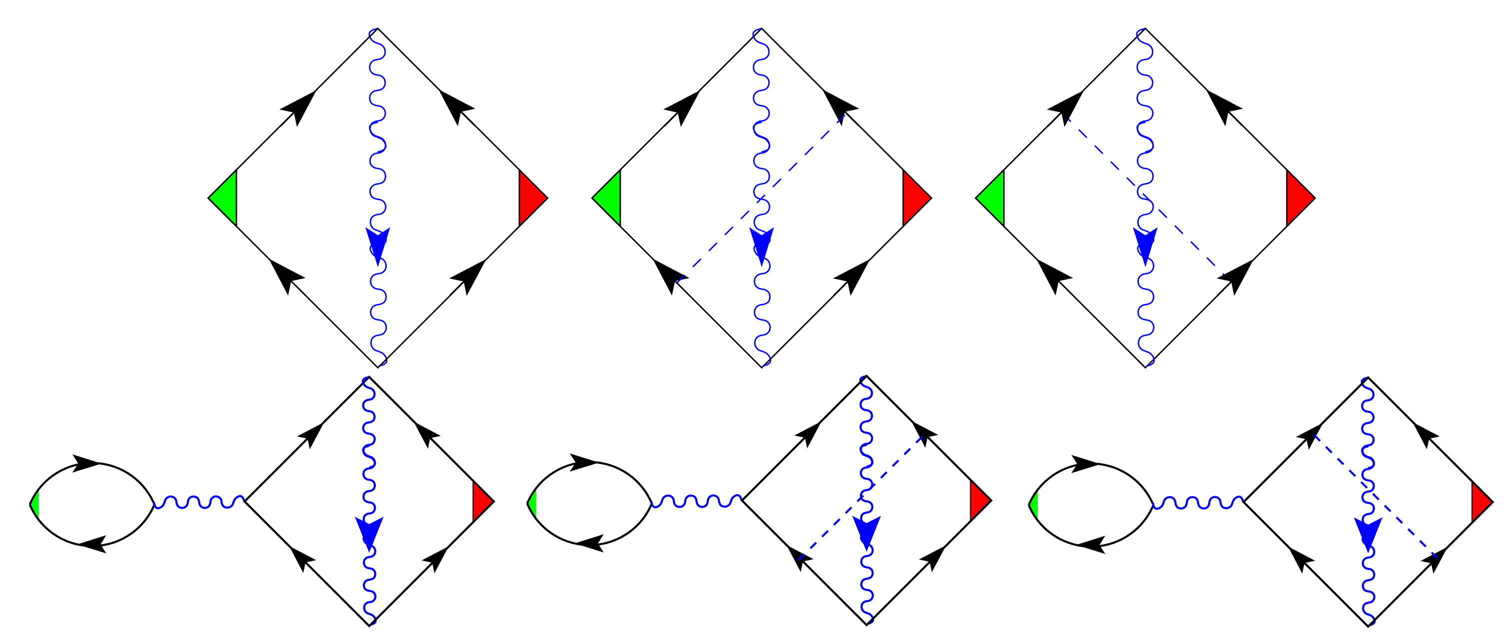
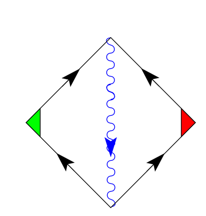


Figure 2. Weak localization diagrams.

An example of the expression for one of the diagrams:



$$= \frac{e}{2\pi m^2}\sum_{\gamma\gamma'=0}^{3}\mathrm{Tr}\left\{\int\frac{d^2p}{(2\pi)^2}G_{\mathrm{A}}(\vec{p})p_y\frac{\sigma^3}{2}G_{\mathrm{R}}(\vec{p})\sigma^\gamma \times \right.$$
$$\left.\times\left[\sigma^{\gamma'}G_{\mathrm{R}}(\vec{q}-\vec{p})\left(-p_x - \frac{e}{c}\tilde{A}_x\right)G_{\mathrm{A}}(-\vec{p})\right]^T\right\}\int\frac{d^2q}{(2\pi)^2}C^{\gamma\gamma'}(\vec{q})$$

All together, the diagrams add up to zero

6 Connection between the spin current and the magnetization

Without magnetic field and for non-magnetic impurities [7]

$$\dot{\hat{s}}_k(t) = -2m\alpha\hat{j}^{s_z}_k(t), \quad k = x, y$$

- valid for arbitrary $\vec{E}$ and for systems with interaction.
The magnetization

$$\langle\dot{\hat{s}}_k\rangle(t) = -2m\alpha\langle\hat{j}^{s_z}_k\rangle(t).$$

In diffusive systems, stationary state is reached at $t \to \infty$. $\Longrightarrow$ the spin-Hall current must be zero at $\omega = 0$.
Thus, the result of [1] for a clean sample $\sigma^z_{yx} = \frac{e}{8\pi}$ means that $\langle\hat{s}_y\rangle \to \infty$, $t \to \infty$, which is impossible!
(this can be also generalised for the Dresselhaus term.)

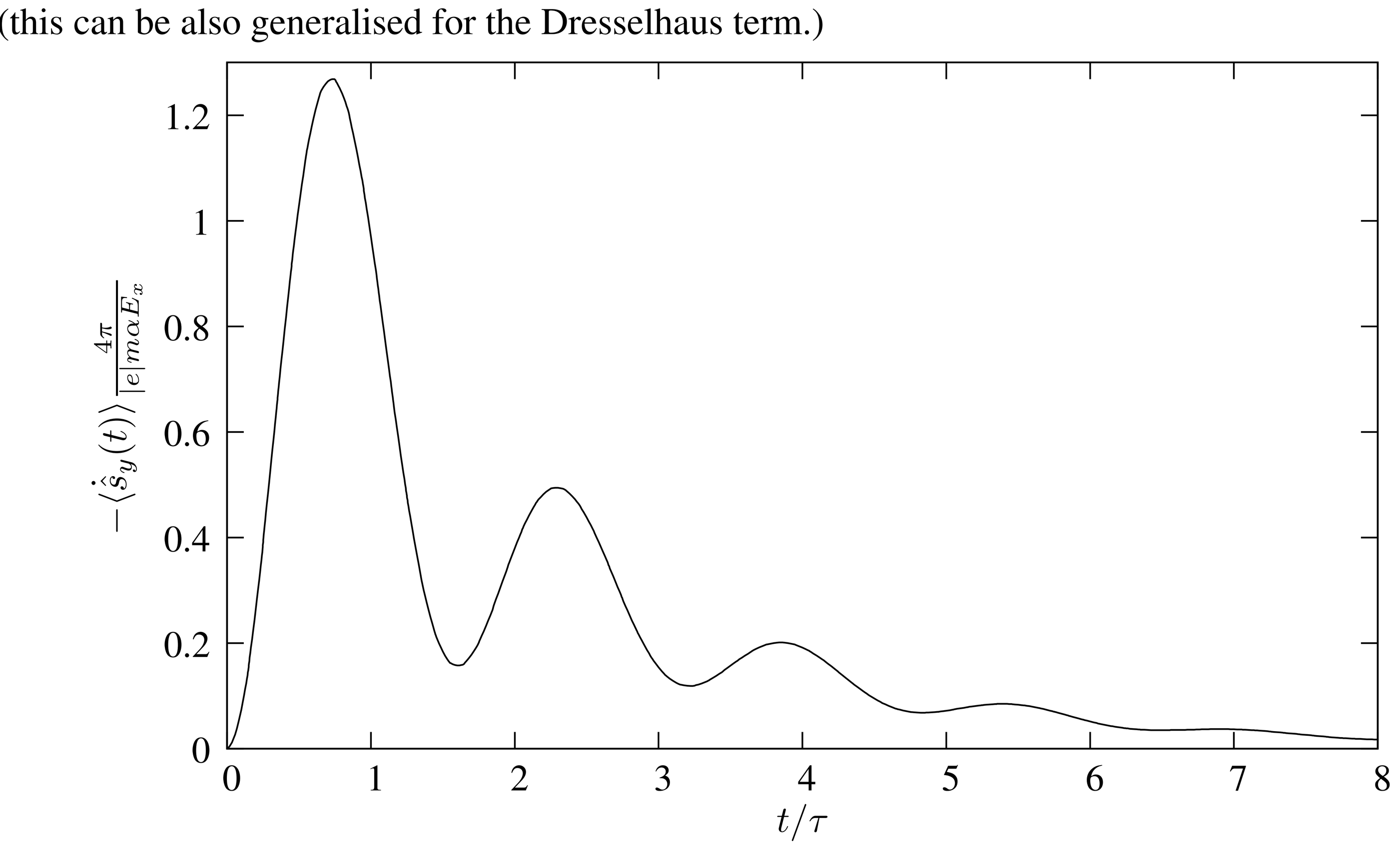


Figure 3. Spin-Hall current decay in time. (only the contribution from zero-loop diagrams) The period of oscillation is $1/\Delta$, and the exponential decay time is τ.

7 Conclusions

- We have calculated the zero-loop and the weak localization contributions to the σ^z_{yx}.
- Both contributions result zero independently.
- General argument: spin-Hall current is zero at $\omega = 0$ and for $B = 0$.

See our article [8].

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