Nonequilibrium persistent currents in mesoscopic disordered systems

Oleg Chalaev

Ph.D. thesis at SISSA

supervisor: Vladimir Kravtsov

24.10.2003

Journal reference:
O. L. Chalaev, V. E. Kravtsov, Phys. Rev. Lett., **89** 17 (176601).

go to conclusions

Thermodynamical vs dynamical equilibrium

Thermodynamical equilibrium:

$$\langle\hat{O}\rangle = \frac{1}{Z}\sum_{\mu} O_{\mu\mu} \exp\left[-\beta E_{\mu}\right]$$

Only diagonal matrix elements are relevant

Dynamical equilibrium (non-equilibrium steady state)

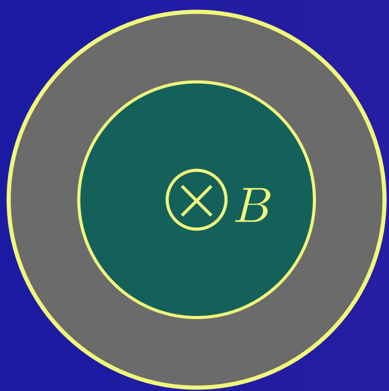
$$\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{\rho}\hat{O}\right] =$$

$$= \sum_{\mu} \rho_{\mu\mu} O_{\mu\mu} + \sum_{\mu\neq\nu} \rho_{\nu\mu} O_{\mu\nu}$$

Off-diagonal matrix elements are also relevant

- When is it relevant for a mesoscopic system?
- How to describe it by the diagrammatic technique?

Persistent current in mesoscopic rings with disorder



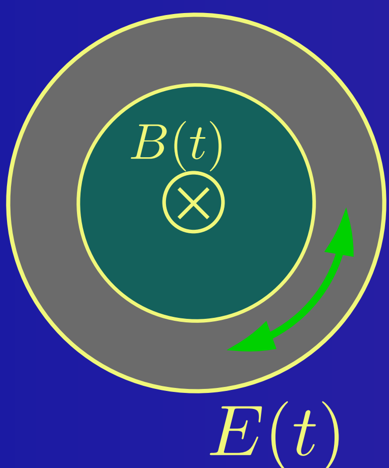
Persistent current in equilibrium:

$$\Phi = \mathrm{const}$$

prediction: L. Grunther & Y. Imry, 1969.

measurement: L. P. Levy et al, 1990;
Chandrasekhar et al, 1991.

calculation: V. Ambegaokar & U. Eckern, 1990.



Time-dependent external force:

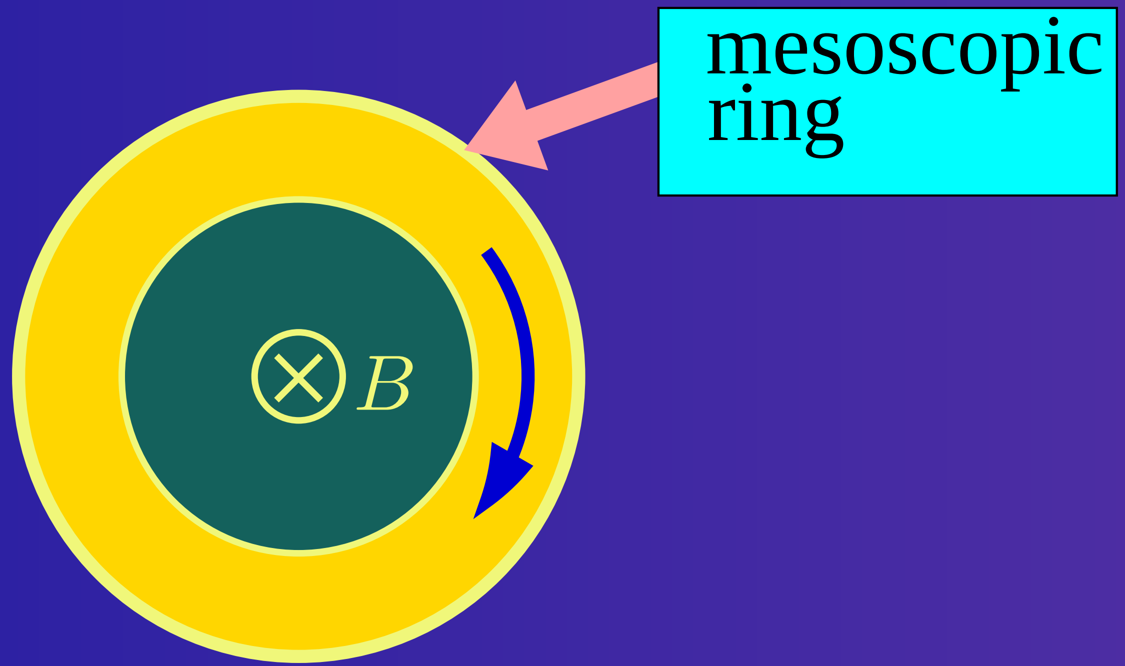
$$\Phi = \Phi_0 + \Phi(t)$$

$\Rightarrow$ rectification : $I_{\mathrm{DC}} \neq 0$

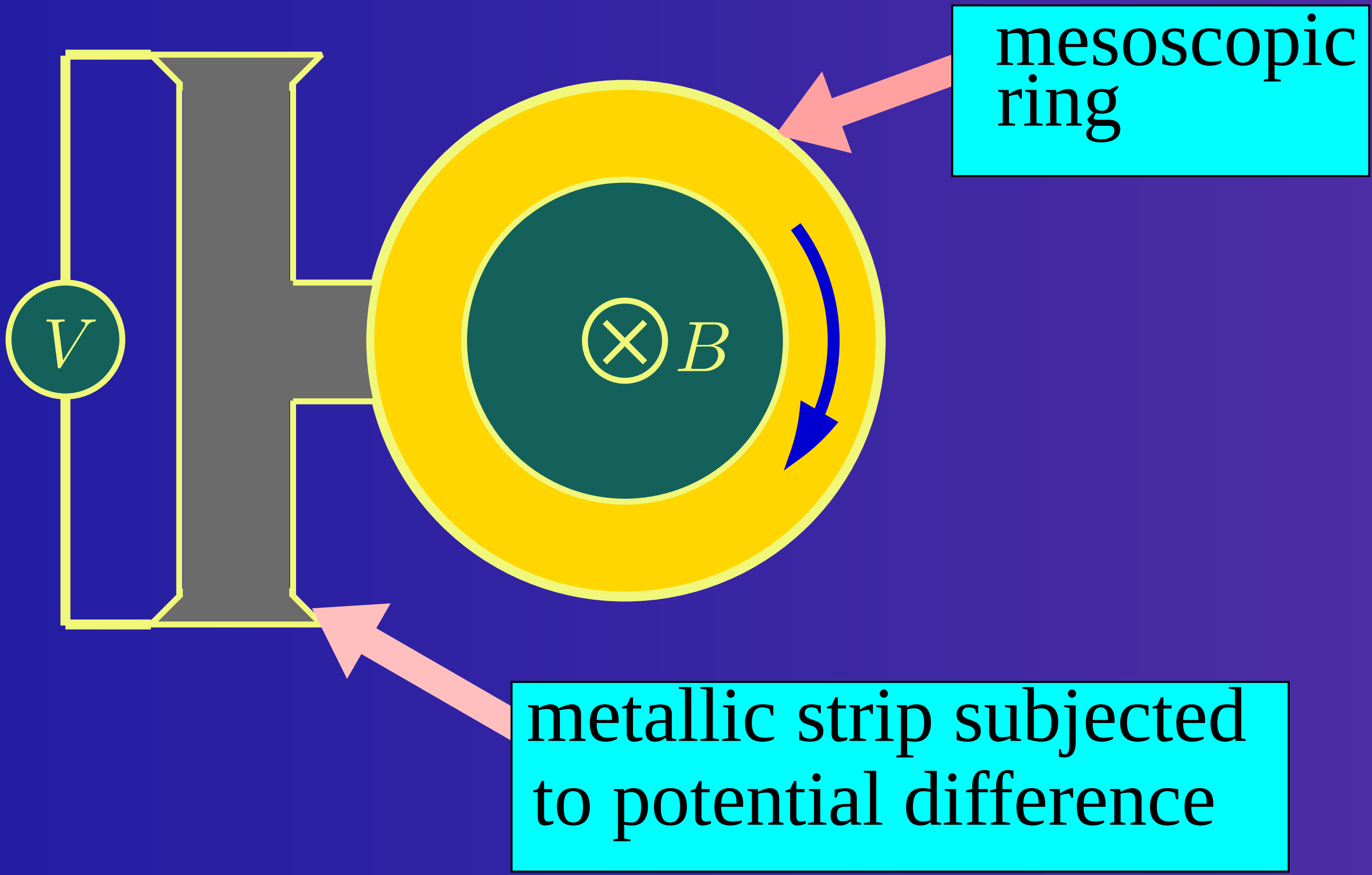
V. E. Kravtsov & V. I. Yudson, 1993;
V. E. Kravtsov & B. L. Altshuler, 2000.

The system is stable because the energy from $\Phi(t)$ is dispersed in the environment (equilibrium reservoir).

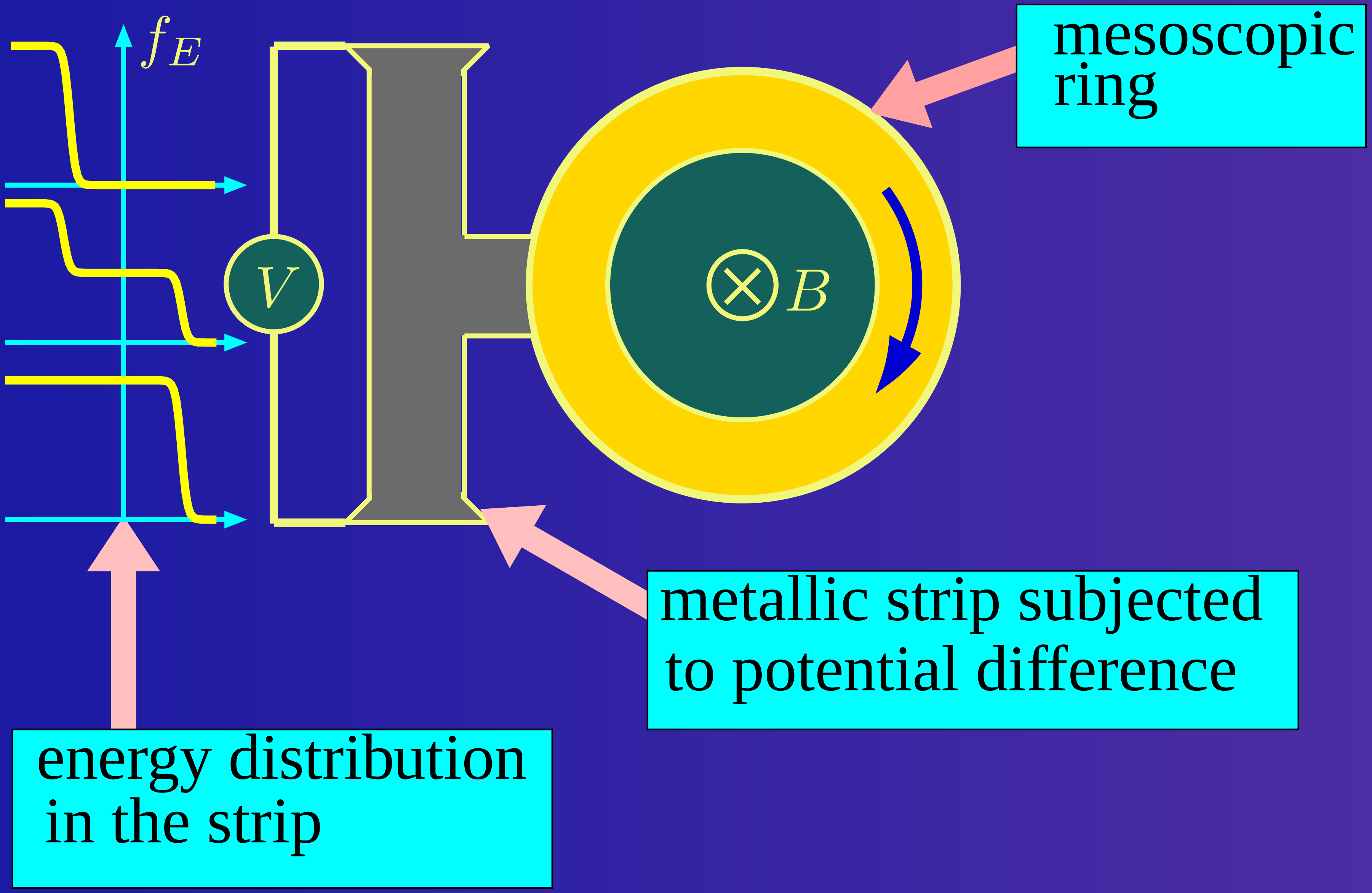
Nonequilibrium established by the reservoir



Nonequilibrium established by the reservoir



Nonequilibrium established by the reservoir



go to slide #14

Perturbation theory in interaction

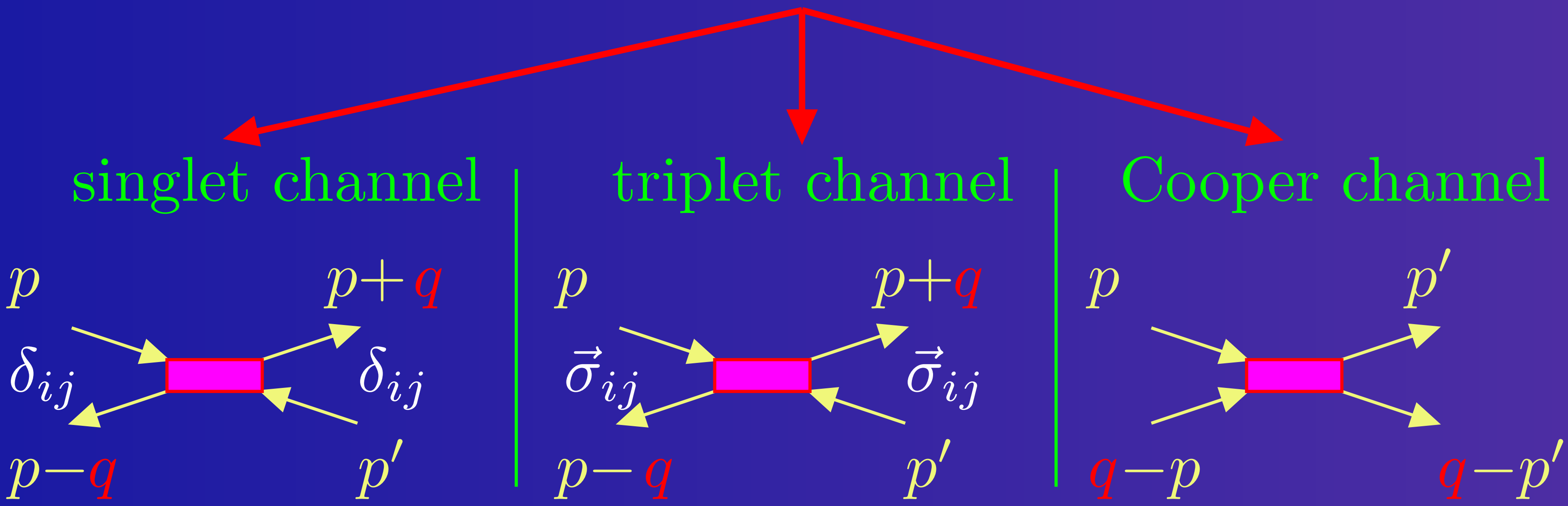
- Non-equilibrium energy distribution function is introduced via ansatz:

$$G_{\mathrm{K}}^{(0)} = h_E \left(G_{\mathrm{R}}^{(0)} - G_{\mathrm{A}}^{(0)} \right), \quad h_E = 1 - 2f_E,$$

where f_E is the energy distribution function.

- We are looking for the diagrams that represent long-range excitations (soft modes).
They correspond to three *excitation channels*.

Excitation channels

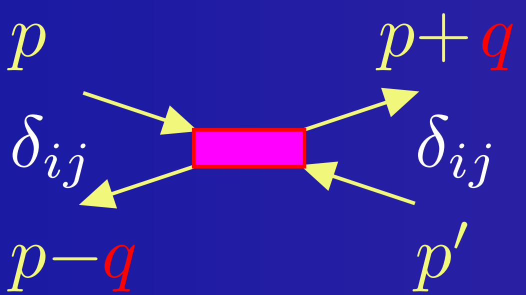


q denotes the (small) momentum of excitations

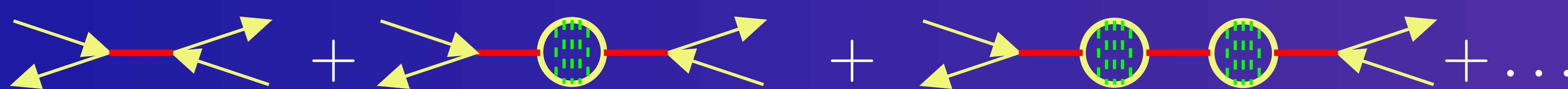
singlet and triplet channel differ by their spin configurations

Excitation channels

singlet channel

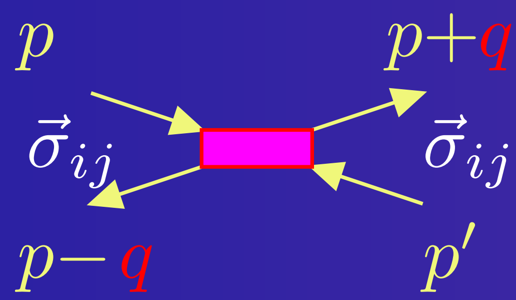


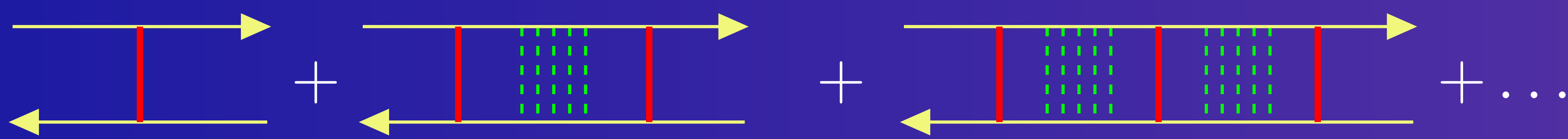
RPA series:



Excitation channels

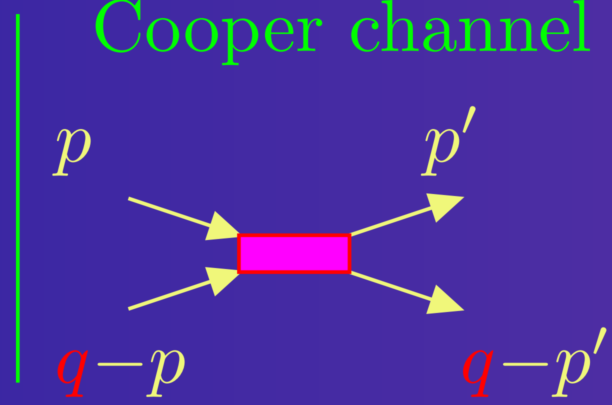
triplet channel



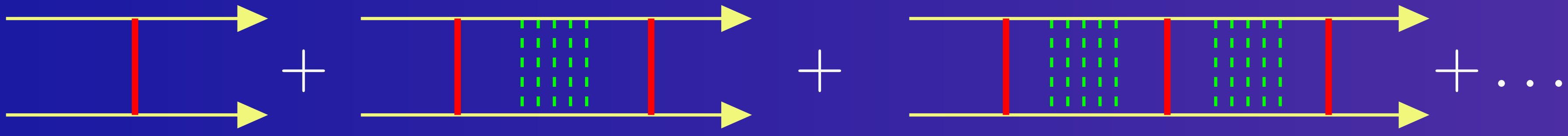


Excitation channels

Cooper channel



Renormalization in the Cooper channel:



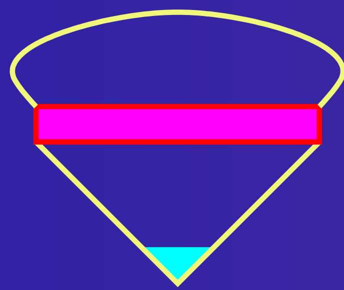
- supressed by the factor $\dfrac{1}{1+\frac{\Lambda}{2}\log\frac{E_F}{T}} \ll 1$

$\Lambda =$ amplitude of the bare Coulomb interaction.

Thermodynamic and kinetic parts of the current in Keldysh technique

If the interaction amplitude is small, we can neglect diagrams with more than one excitation channel:

$$j = \mathrm{Tr}[\hat{j} G_{\mathrm{K}}] =$$

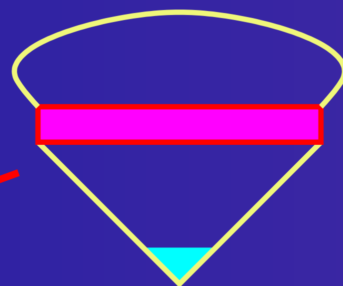


$$= j' + j'',$$

Thermodynamic and kinetic parts of the current in Keldysh technique

If the interaction amplitude is small, we can neglect diagrams with more than one excitation channel:

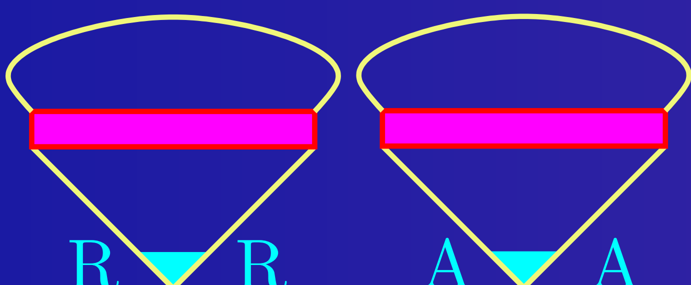
$$j = \mathrm{Tr}[\hat{j} G_{\mathrm{K}}] =$$



$$= j' + j'',$$

thermodynamic part

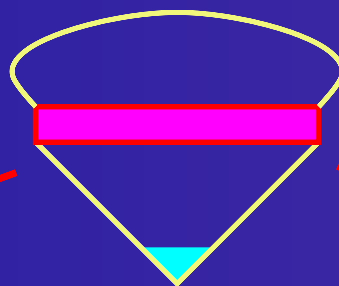
$$j' = \mathrm{Tr}[\hat{j} h_E (G_{\mathrm{R}} - G_{\mathrm{A}})]$$



Thermodynamic and kinetic parts of the current in Keldysh technique

If the interaction amplitude is small, we can neglect diagrams with more than one excitation channel:

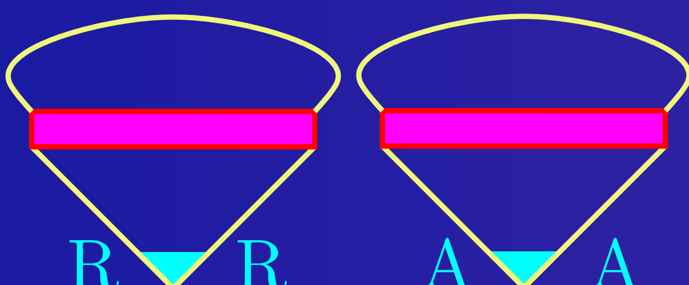
$$j = \mathrm{Tr}[\hat{j} G_{\mathrm{K}}] =$$



$$= j' + j'',$$

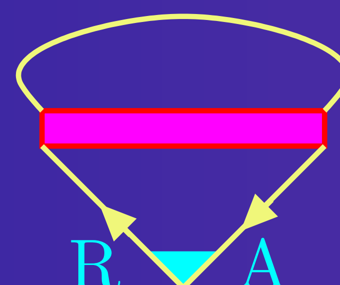
thermodynamic part

$$j' = \mathrm{Tr}[\hat{j} h_E (G_{\mathrm{R}} - G_{\mathrm{A}})]$$



kinetic part

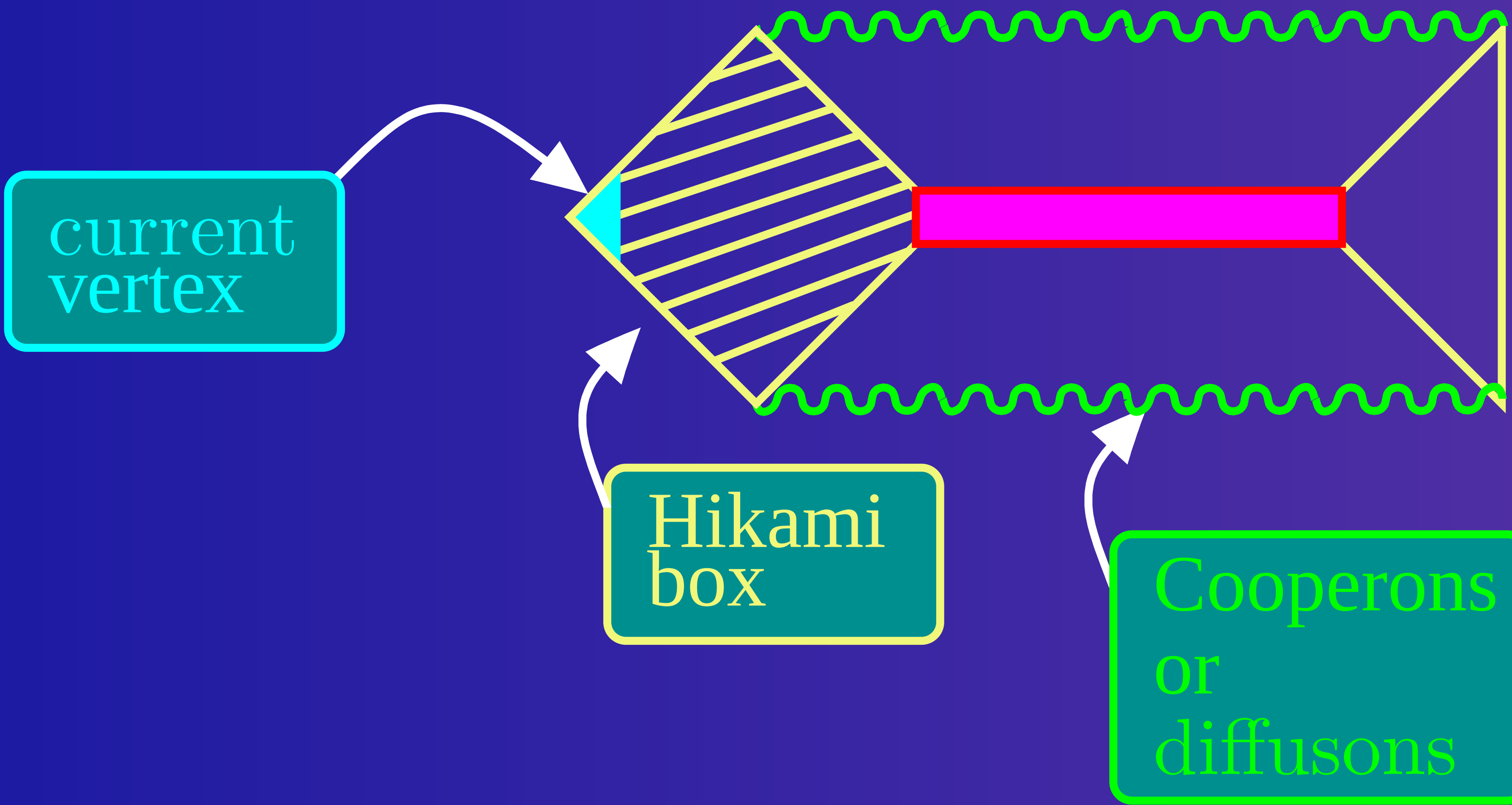
$$j'' = \mathrm{Tr}[\hat{j} \{G_{\mathrm{K}} - h_E (G_{\mathrm{R}} - G_{\mathrm{A}})\}]$$



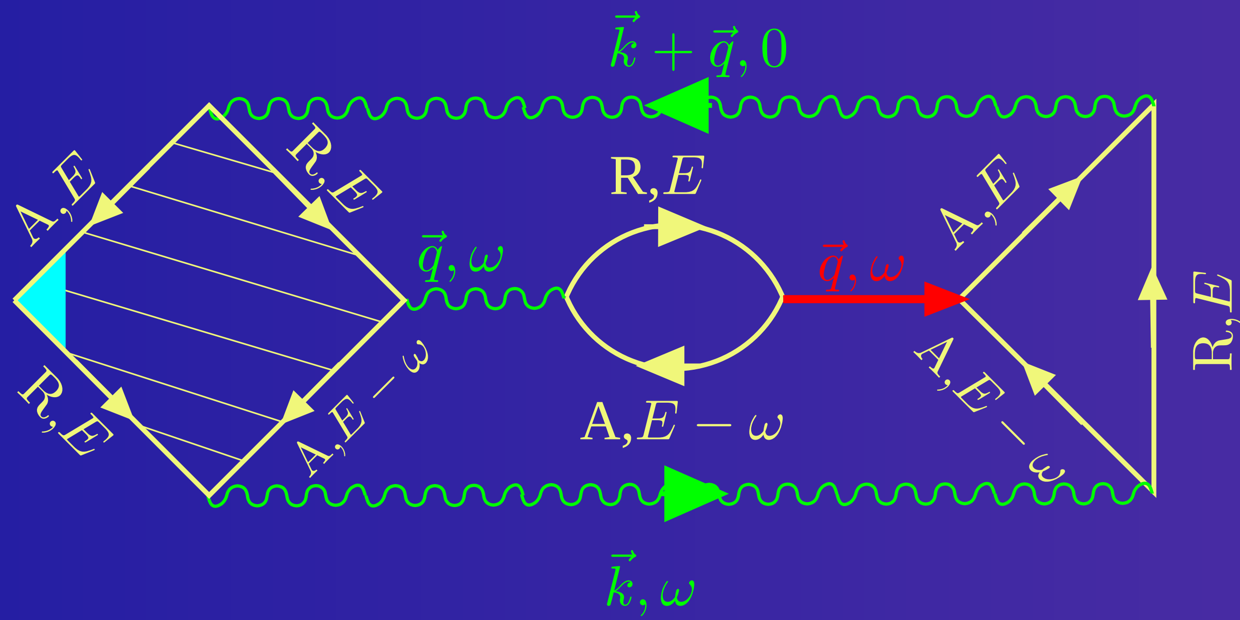
=0 in equilibrium

After the averaging over disorder

After adding external cooperons and diffusons, all three channels are represented by one and the same diagram:



The singlet channel



the effective interaction is equal to

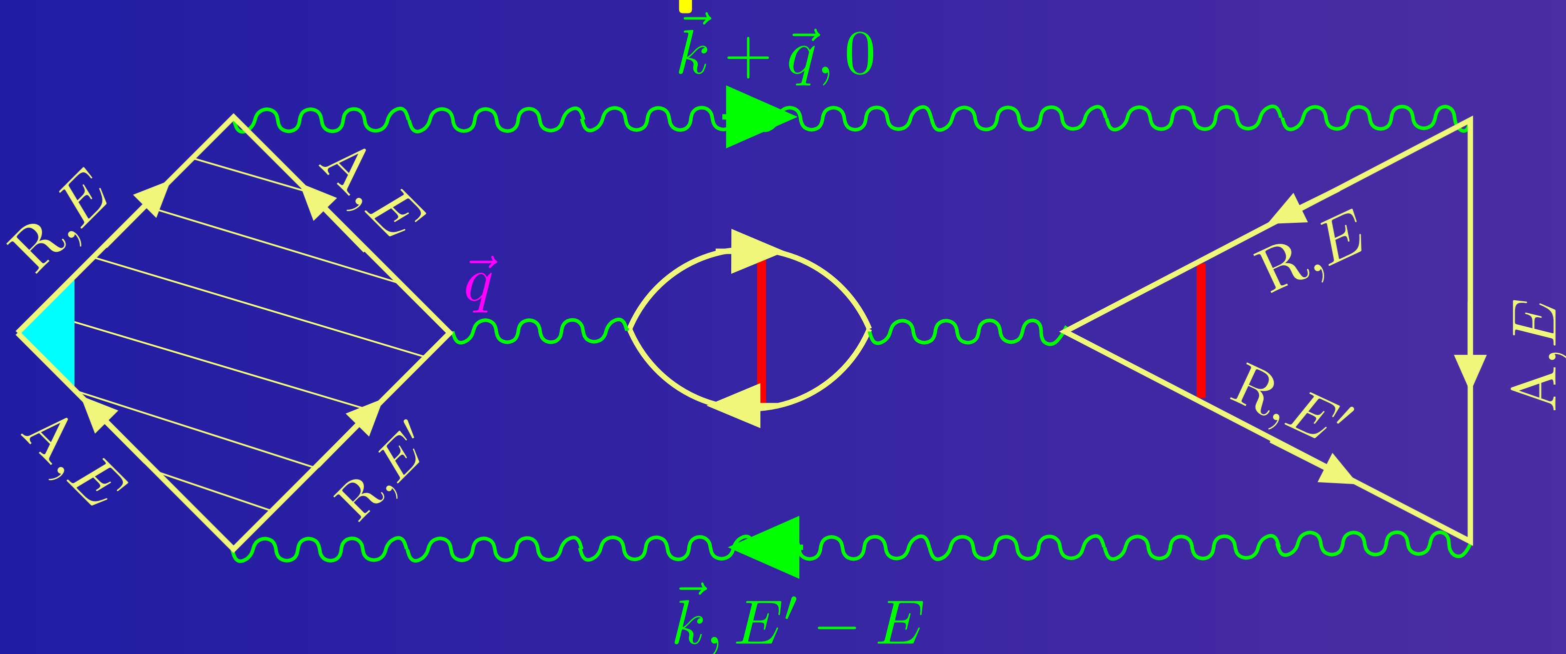
$$\frac{1}{2\nu_0 D_0 q^2}\int \mathrm{d}E' R_\omega(E,E')\frac{(D_0q^2)^2+\omega^2}{(D_{E'}q^2)^2+\omega^2},$$

ν_E =density of states, $D_E = v^2\tau_0\nu_0/(3\nu_E)$ =diffusion coeff.

$$R_\omega(E,E') = (h_E - h_{E-\omega})(1-h_{E'}h_{E'-\omega}) - (E \leftrightarrow E').$$

In equilibrium $h_E = \tanh\frac{E}{2T}$ so that $R_\omega(E,E') = 0$.

The triplet channel



The two bare interaction lines provide the coefficient

$$\frac{\Lambda^2}{4\nu_0^2}R_\omega(E,E')$$

with the same $R_\omega(E,E')$ as in the singlet channel.

Connection to the inelastic collision integral:

$$St[E] = \int \mathrm{d}E' \mathrm{d}\omega P(\omega) R_\omega(E, E')$$

The global balance condition:

$$\int \mathrm{d}E St[E] = 0$$

follows from

$$\int \mathrm{d}E \mathrm{d}E' R_\omega(E, E') = 0$$

$\Longleftrightarrow$ for constant density of states $j'' = 0$.

Result

Relaxation-induced (averaged) current:

$$I^{(\mathrm{r})} = \sum_{n\geq 1} \sin\left[4\pi n \frac{\Phi}{\Phi_0}\right] I_n^{(r)},$$

where

$$I_n^{(r)} = -\frac{e}{3hg}(1-3\Lambda^2)\int \mathrm{d}E \left(\frac{\delta D_E}{D_0}\right)\left[\tilde{T}\frac{\partial f_E}{\partial E} + f_E(1-f_E)\right]$$

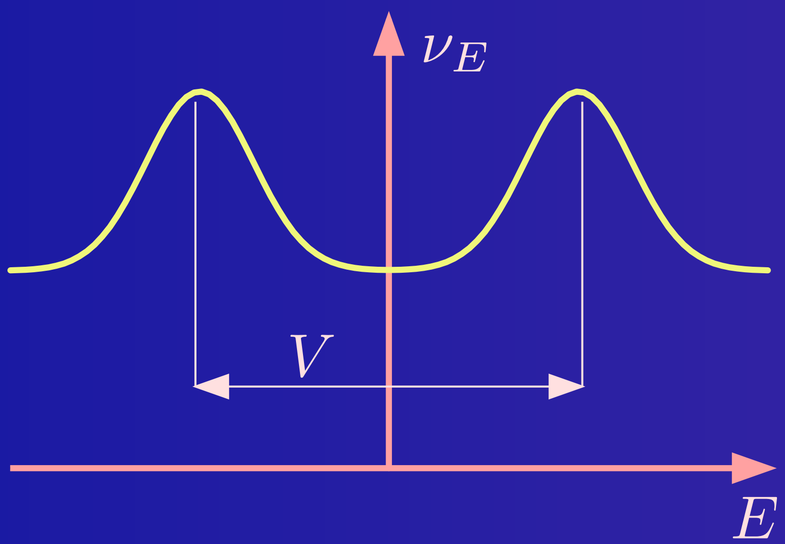
$g = \nu DS/L$ is the dimensionless conductance

Λ is the amplitude of the bare Coulomb interaction

$\tilde{T} = \int \mathrm{d}E\, f_E\,(1-f_E)$ is the effective temperature; $\tilde{T} \gtrsim \frac{V}{4}$.

D_E dependence from Kondo effect

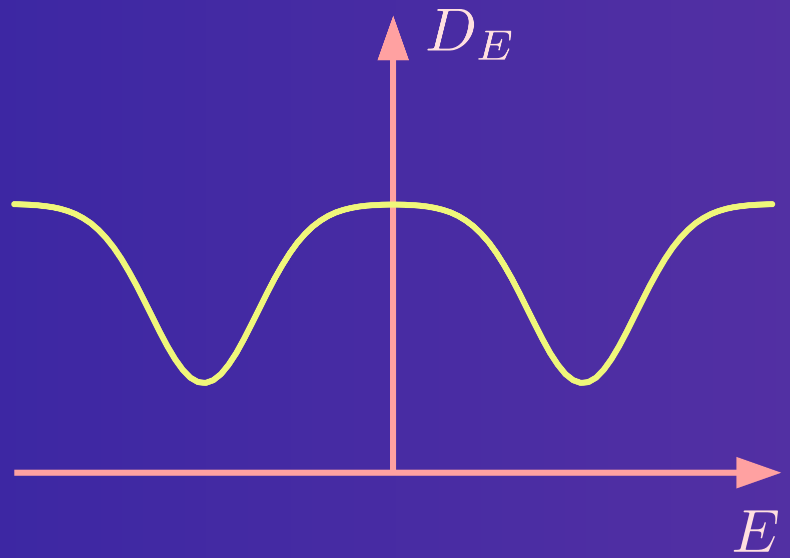
Density of states

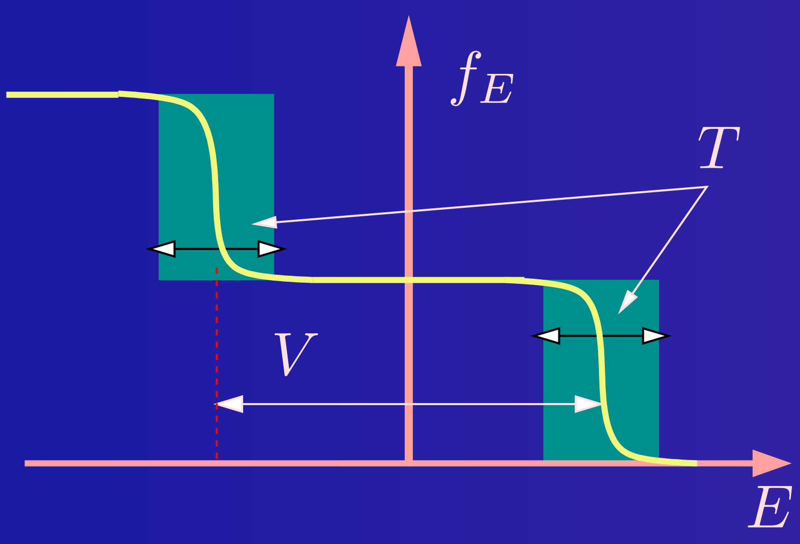


$\nu_E D_E = \text{const}$

$\Longleftrightarrow$

Diffusion coefficient





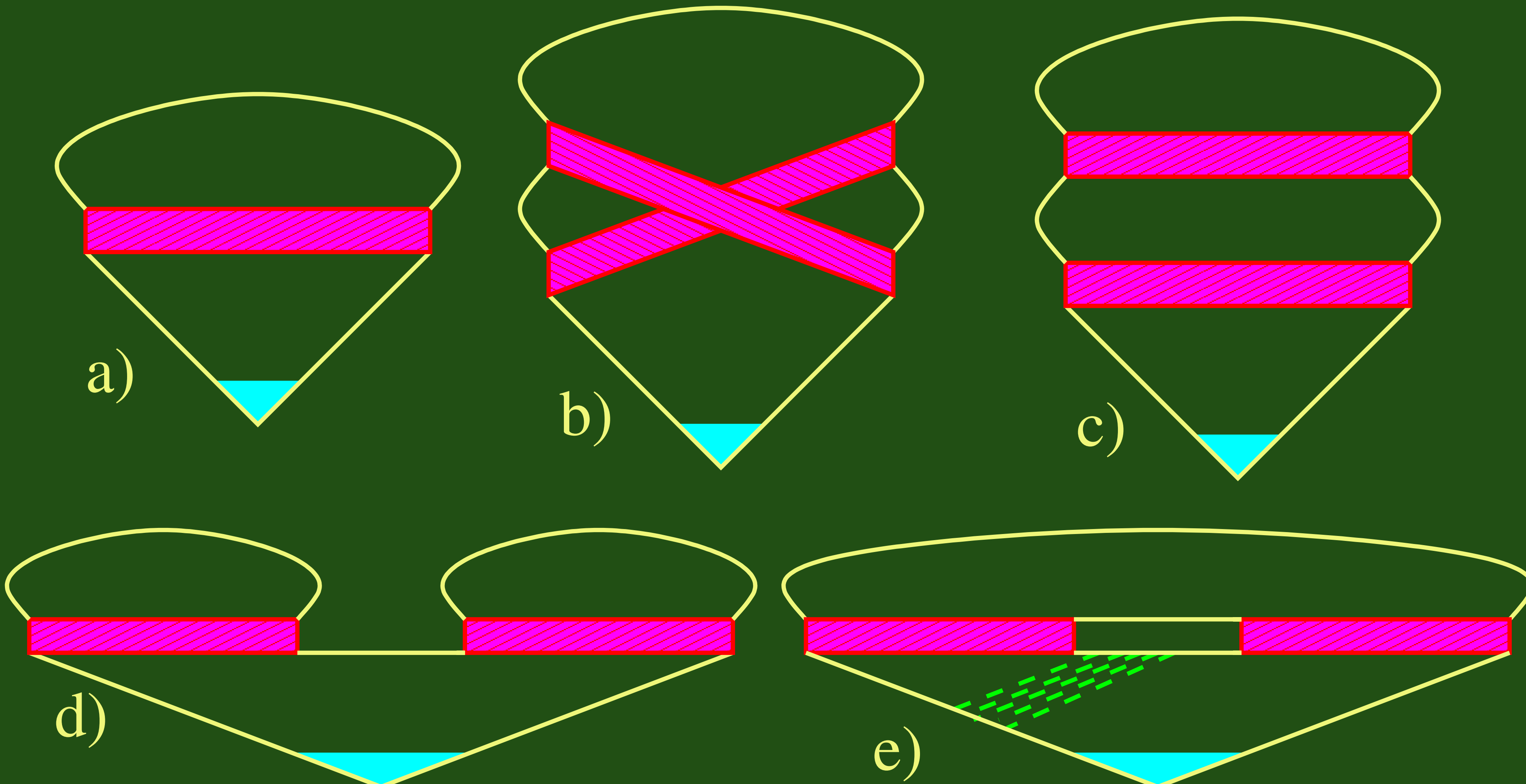
go to slide #4

	Thermodynamical current	Kinetic current
amplitude	$0.1\mathrm{nA} \times \log^{-1}\left[\frac{E_F}{E_\mathrm{T}}\right]$, $\left(0.1\mathrm{nA} = \frac{eE_\mathrm{T}}{\hbar}\right)$	$0.1\mathrm{nA} \times \frac{T_\mathrm{K}}{E_\mathrm{T}} \times \frac{1}{g} \times \frac{\delta D}{D}$, $\frac{\delta D}{D} \sim \frac{n_K}{n_0} \times \frac{l}{\lambda_F}$
temperature dependence	$\exp\left[-\frac{T}{E_\mathrm{T}}\right]$, $E_\mathrm{T} \sim 10^{-2} K$	independent of the bath temperature T if $T \ll V$

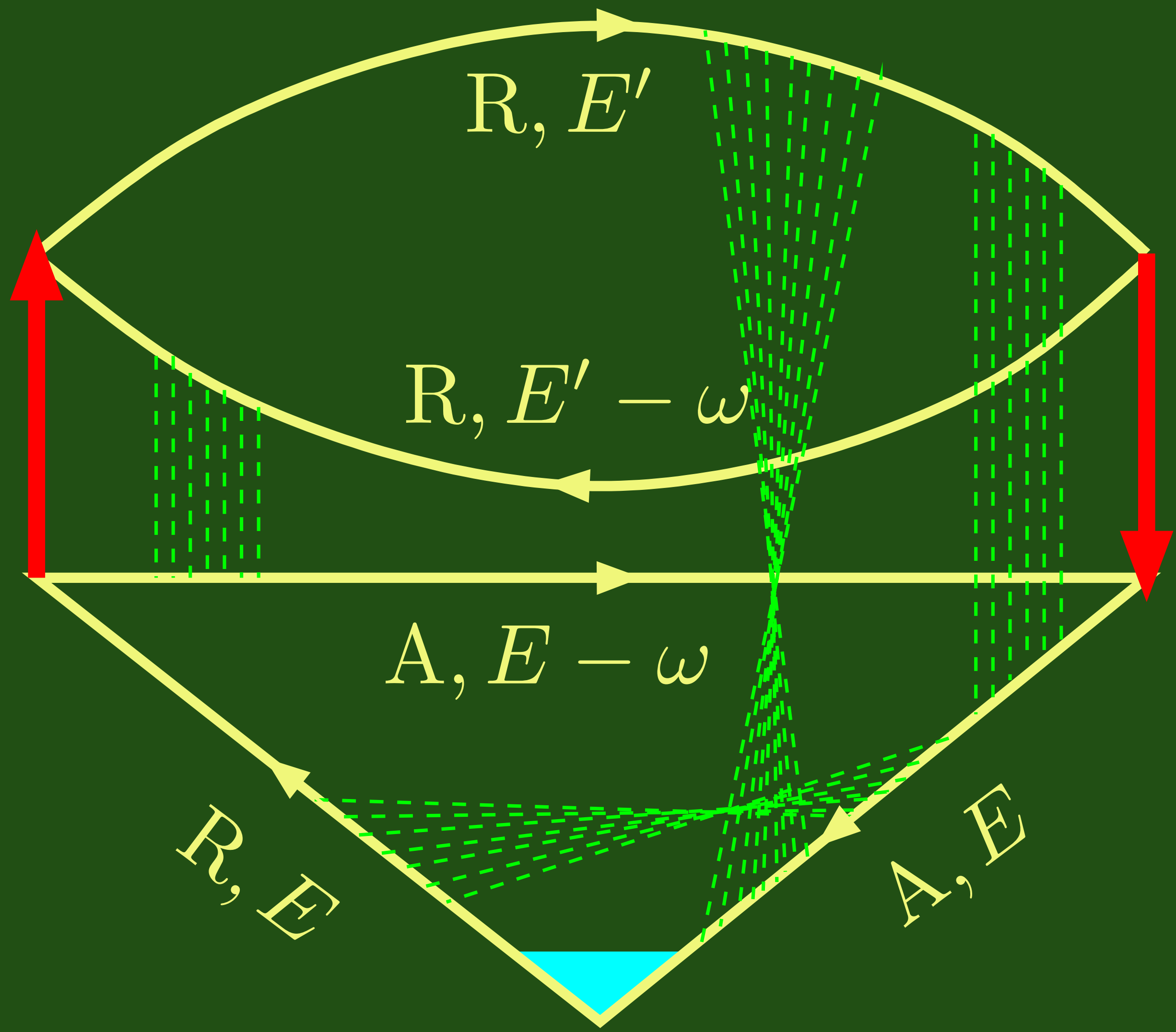
Conclusions

- In the presence of interaction and at a nonequilibrium electron energy distribution there is a kinetic contribution to the current (magnetization) related to relaxation.
- The disorder average of the relaxation-induced current is zero unless the diffusion coefficient is energy dependent.
- In contrast to the equilibrium persistent current, the relaxation-induced current is not exponentially small if the effective temperature is much larger than the Thouless energy.

To be shown upon request



The triplet channel



The superconducting channel

The contribution of the superconducting channel is supressed by the factor of

$$\frac{1}{1+\frac{\Lambda}{2}\log\frac{E_{\mathrm{F}}}{T}} \times \frac{E_{\mathrm{T}}^2}{T^2} \ll 1,$$

and we neglect it.

$E_{\mathrm{T}} =$ Thouless energy.