# Magnetization of mesoscopic rings in a non-equilibrium steady state

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Journal reference:

O. L. Chalaev, V. E. Kravtsov, Phys. Rev. Lett., **89** 17 (176601).

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# Thermodynamical vs dynamical equilibrium

Thermodynamical equilibrium:

$$\langle\hat{O}\rangle = \frac{1}{Z}\sum_{\mu} O_{\mu\mu}\exp\left[-\beta E_{\mu}\right]$$

Only diagonal matrix elements are relevant

Dynamical equilibrium:

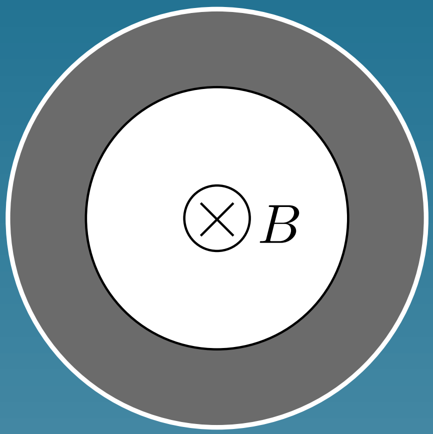
$$\langle\hat{O}\rangle = \mathrm{Tr}\left[\hat{\rho}\hat{O}\right] =$$

$$= \sum_{\mu}\rho_{\mu\mu}O_{\mu\mu} + \sum_{\mu\neq\nu}\rho_{\nu\mu}O_{\mu\nu}$$

Off-diagonal matrix elements are also relevant

- When is it relevant for a mesoscopic system?
- How to describe it by the diagrammatic technique?

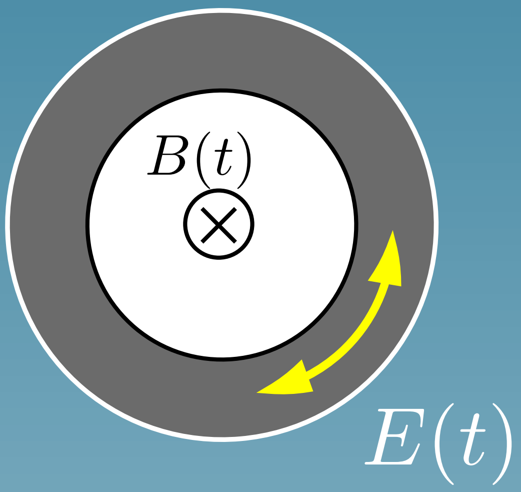
# Current in a mesoscopic ring with disorder: a bit of history




persistent current in equilibrium:

$$\Phi = \text{const}$$

V. Ambegaokar & U. Eckern, 1990.




Time-dependent external force:

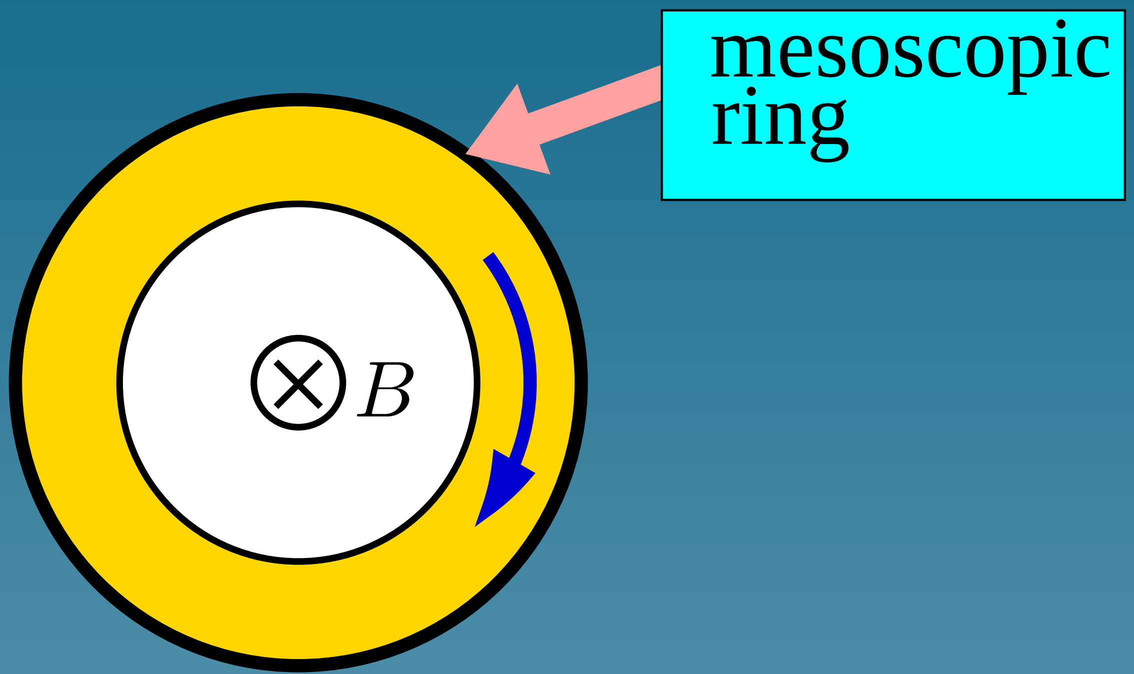
$$\Phi = \Phi_0 + \Phi(t)$$

$\Rightarrow$ rectification : $I_{\rm DC} \neq 0$

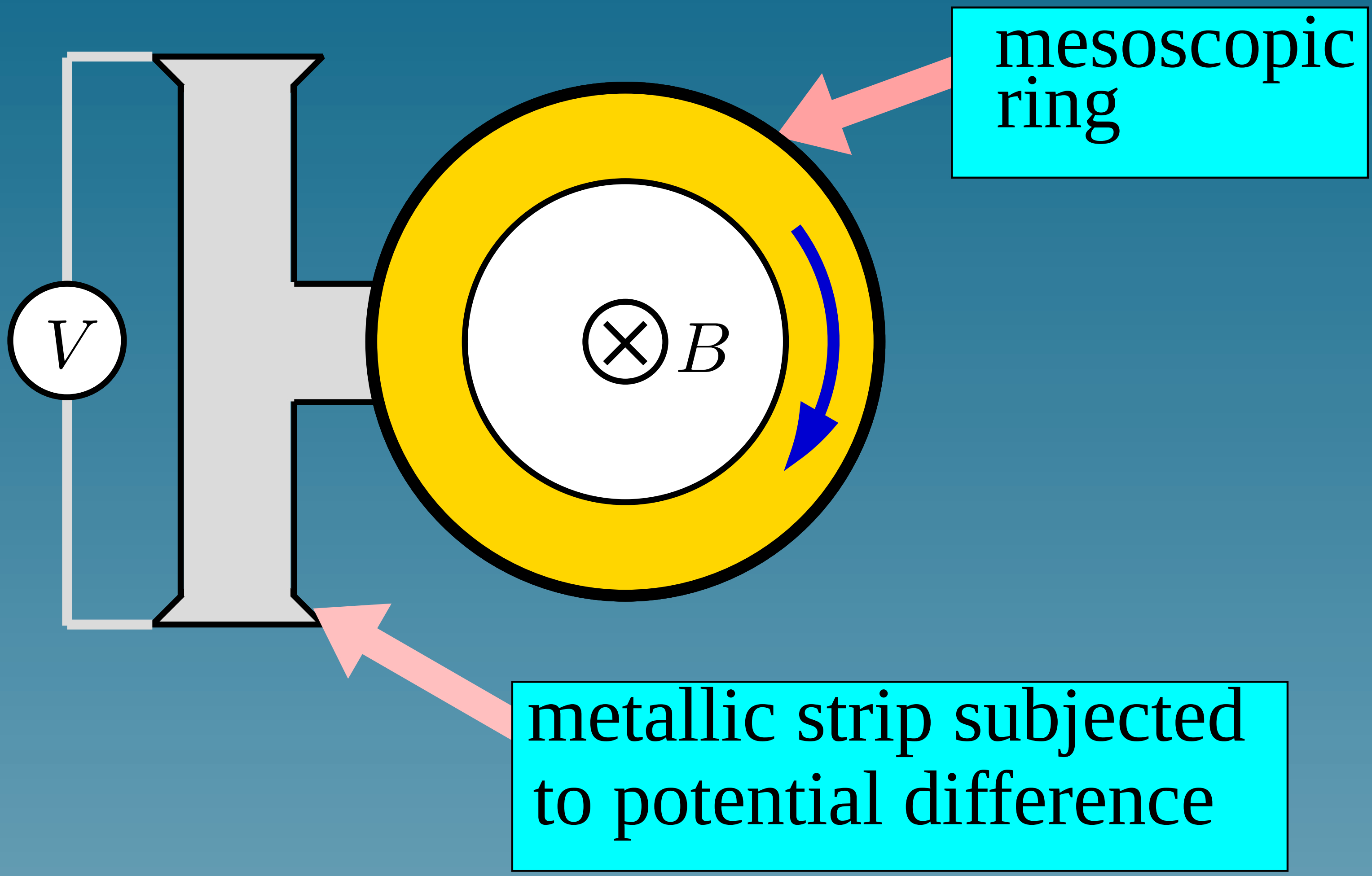
V. E. Kravtsov & B. L. Altshuler, 2000.

V. E. Kravtsov & V. I. Yudson, 1993.

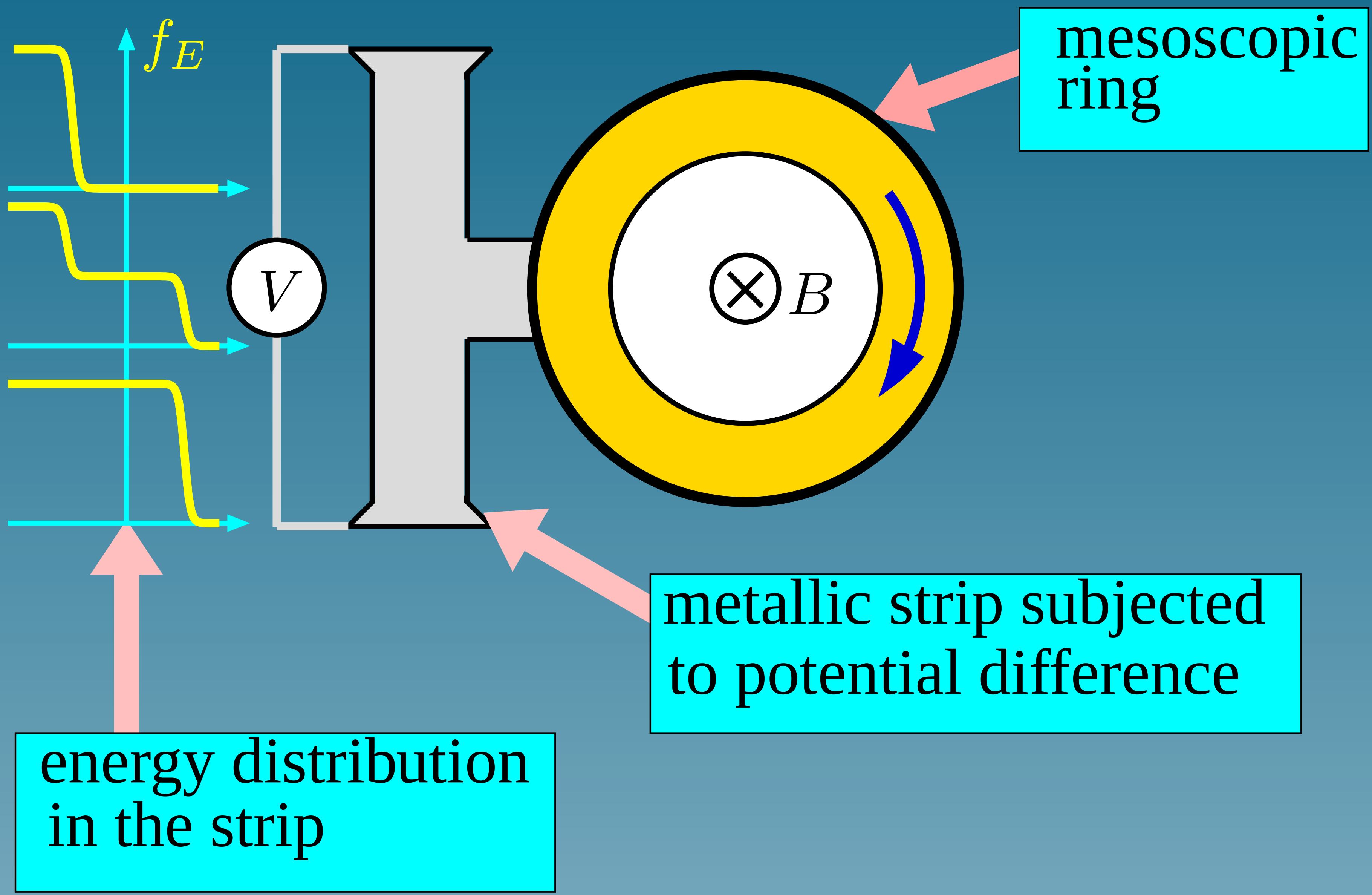
# Possible realization



# Possible realization

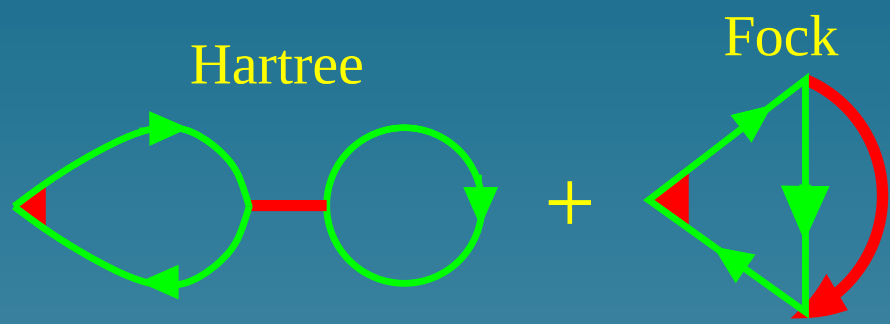


# Possible realization



# Perturbation theory

$\delta G = -$



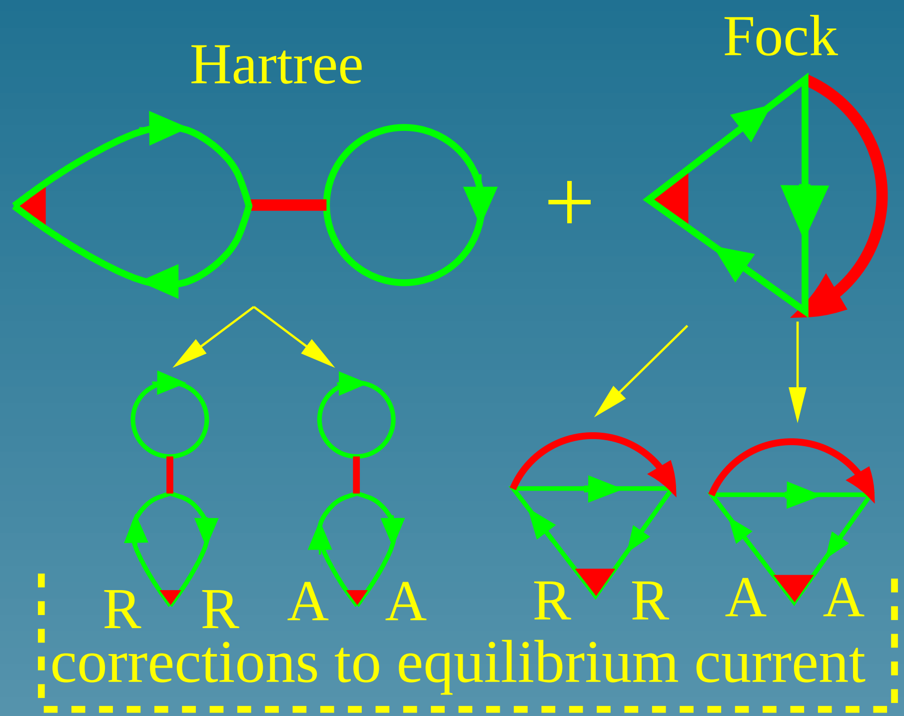

Non-equilibrium energy distribution function is introduced via ansatz:

$$G_{\mathrm{K}}^{(0)} = h_E \Big( G_{\mathrm{R}}^{(0)} - G_{\mathrm{A}}^{(0)} \Big), \quad h_E = 1 - 2f_E$$

$$\text{current} = \hat{\vec{j}} G_{\mathrm{K}}$$

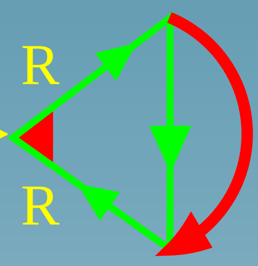
# Perturbation theory

$\delta G = -$

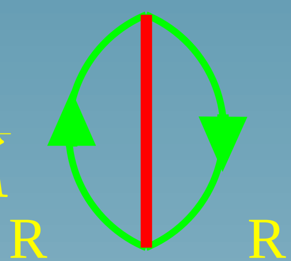



Thermodynamic current:

$\vec{j} = -\frac{\partial \Omega}{\partial \vec{A}} \Leftrightarrow$



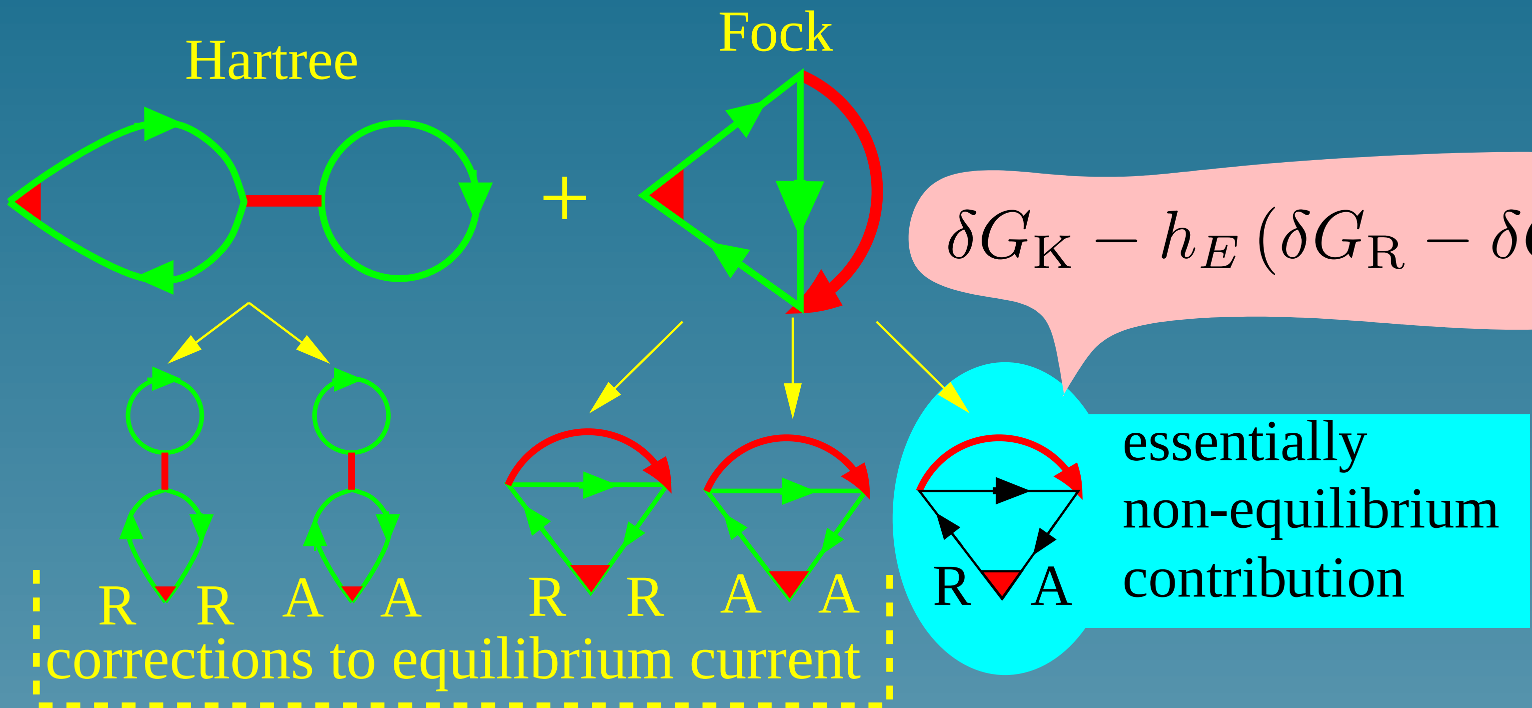

$\propto \frac{\partial}{\partial \vec{A}}$



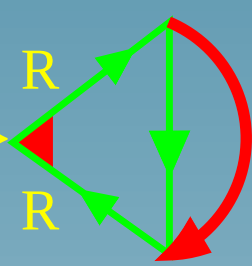
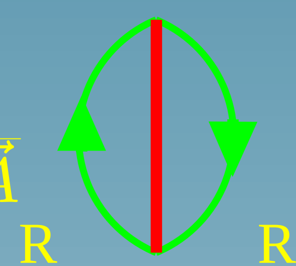

, etc.

# Perturbation theory

$\delta G = -$




Thermodynamic current:

$\vec{j} = -\dfrac{\partial \Omega}{\partial \vec{A}} \Leftrightarrow$  $\propto \dfrac{\partial}{\partial \vec{A}}$  , etc.

# Before averaging over disorder

Thermodynamic contribution:

$$I^{(eq)} = A\mathbf{RR}\,(U_{\rm R}^{\omega} - 2U_{\rm R}^{0})\,(1 - h_E h_{E-\omega}) +$$

$$+R\mathbf{AA}\,(U_{\rm A}^{\omega} - 2U_{\rm A}^{0})\,(1 - h_E h_{E-\omega}) + (R\mathbf{RR} - A\mathbf{AA})h_E U_{\rm K}^{\omega} -$$
$$-(1 - h_E h_{E-\omega})[R\mathbf{RR}\,(U_{\rm R}^{\omega} - 2U_{\rm R}^{0}) + A\mathbf{AA}\,(U_{\rm A}^{\omega} - 2U_{\rm A}^{0})],$$

Kinetic contribution:

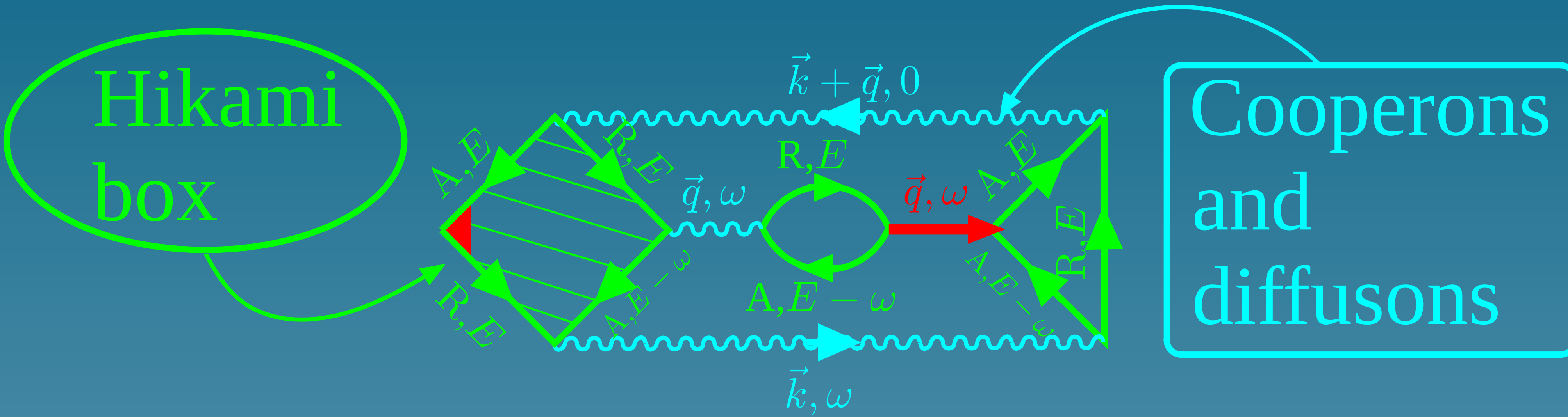
$$I^{(r)} = (\mathrm{A}\mathbf{RA} - \mathrm{R}\mathbf{RA})[(h_E - h_{E-\omega})U_{\rm K} -$$

$$-(1 - h_E h_{E-\omega})(U_{\rm R} - U_{\rm A})]$$

If one substitutes $\hat{\vec{j}} \to \hat{1}$, $\Longrightarrow I^{(r)} \to 0$

$\Longleftarrow I^{(r)}$ is represented by off-diagonal matrix elements.

# After the averaging over disorder




the effective interaction: $\frac{1}{2\nu_0 D_0 q^2}\int \mathrm{d}E' R_\omega(E,E')\frac{(D_0q^2)^2+\omega^2}{(D_{E'}q^2)^2+\omega^2}$,

$\nu_E =$density of states, $D_E = v^2\tau_0\nu_0/(3\nu_E) =$diffusion coeff.

$R_\omega(E,E') =$

$(h_E - h_{E-\omega})(1-h_{E'}h_{E'-\omega}) - (h_{E'}-h_{E'-\omega})(1-h_E h_{E-\omega})$

In equilibrium $h_E = \tanh\frac{E}{2T}$ so that $R_\omega(E,E') = 0$.

# Connection to the inelastic collision integral:

$$St[E] = \int \mathrm{d}E' \mathrm{d}\omega P(\omega) R_\omega(E, E')$$

The global balance condition:

$$\int \mathrm{d}E St[E] = 0$$

follows from

$$\int \mathrm{d}E \mathrm{d}E' R_\omega(E, E') = 0$$

$\Longleftrightarrow$ for constant density of states one gets zero result.

# Result of the calculations

Relaxation-induced (averaged) current:

$$I^{(\mathrm{r})} = \sum_{n\geq 1} \sin\left[4\pi n \frac{\Phi}{\Phi_0}\right] I_n^{(r)},$$

where

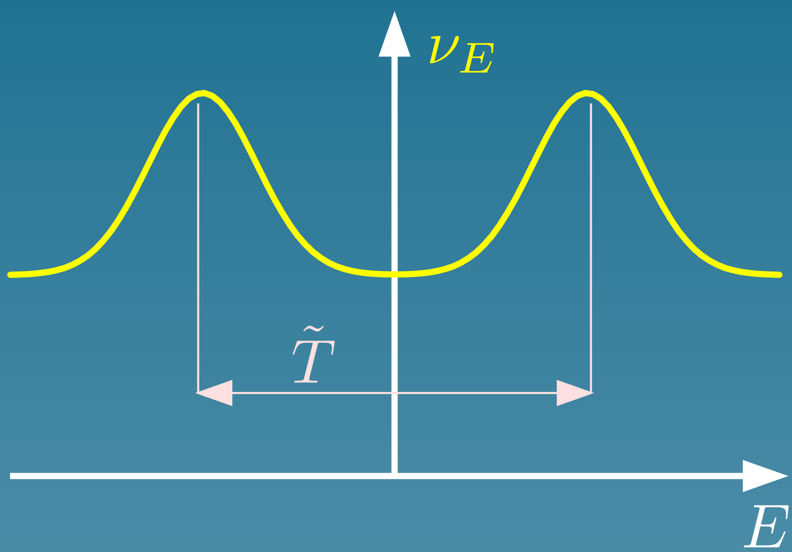
$$I_n^{(r)} = -\frac{C_n e}{3hg} \int \mathrm{d}E \left(\frac{\delta D_E}{D_0}\right) \left[\tilde{T}\frac{\partial f_E}{\partial E} + f_E(1 - f_E)\right]$$

$g = \nu DS/L$ is the dimensionless conductance

$\tilde{T} = \int \mathrm{d}E\, f_E\,(1 - f_E)$ is the effective temperature.

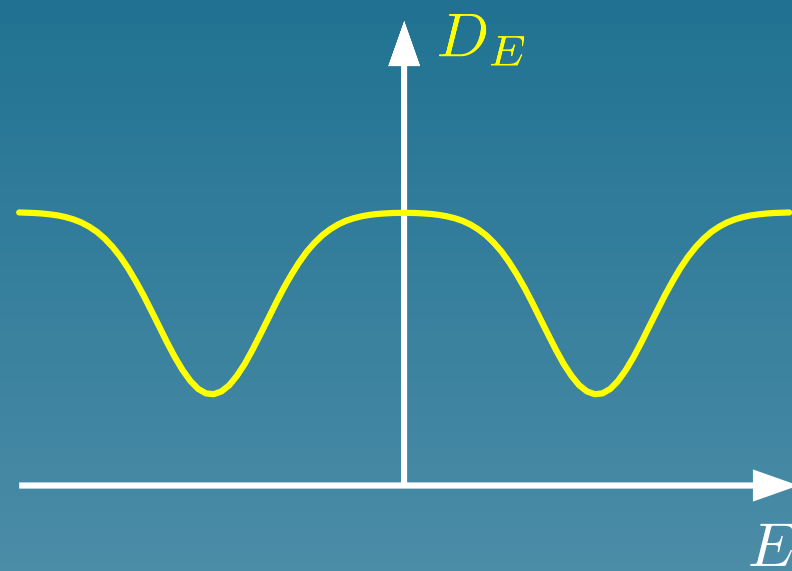
# $D_E$ dependence from Kondo effect

Density of states




$$\nu_E D_E = \text{const}$$
$$\Longleftrightarrow$$

Diffusion coefficient




effective temperature $= \tilde{T} \sim V$ =voltage on the strip

| | Thermodynamical current | Non-equilibrium current |
|---|---|---|
| amplitude | $0.1\mathrm{nA} \times \log^{-1}\left[\frac{E_F}{E_T}\right]$ | $100\mathrm{nA} \times \frac{1}{g} \times \frac{\delta D}{D}$, $\frac{\delta D}{D} \sim \frac{n_K}{n_0} \times \frac{l}{\lambda_F}$ |
| temperature dependence | $\exp\left[-\frac{\tilde{T}}{E_T}\right]$, $E_T \sim 10^{-2} K$ | $\exp\left[-\frac{1}{\sqrt{E_T \tau_\phi}}\right]$, $\frac{1}{\sqrt{\tau_\phi}} \propto \tilde{T}^{1/3}$ |

# Conclusions

- In the presence of interaction and at a nonequilibrium electron energy distribution there is a kinetic contribution to the current (magnetization) related to relaxation.

- The disorder average of the relaxation-induced current is zero unless the diffusion coefficient is energy dependent.

- In contrast to the equilibrium persistent current, the relaxation-induced current is not exponentially small if the effective temperature is much larger than the Thouless energy.

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