Magnetization of mesoscopic rings in a non-equilibrium steady state

Oleg Chalaev, SISSA (Trieste, Italy) in collaboration with Vladimir Kravtsov


Thanks to Igor Aleiner and Boris Altshuler for helpful discussions.
Thermodynamical vs dynamical equilibrium

Thermodynamical equilibrium:

\[ \langle \hat{O} \rangle = \frac{1}{Z} \sum_{\mu} O_{\mu\mu} \exp \left[ -\beta E_{\mu} \right] \]

Only diagonal matrix elements are relevant

Dynamical equilibrium:

\[ \langle \hat{O} \rangle = \text{Tr} \left[ \hat{\rho} \hat{O} \right] = \sum_{\mu} \rho_{\mu\mu} O_{\mu\mu} + \sum_{\mu \neq \nu} \rho_{\nu\mu} O_{\mu\nu} \]

Off-diagonal matrix elements are also relevant

- When is it relevant for a mesoscopic system?
- How to describe it by the diagrammatic technique?
Current in a mesoscopic ring with disorder: a bit of history

persistent current in equilibrium:

\[ \Phi = \text{const} \]


Time-dependent external force:

\[ \Phi = \Phi_0 + \Phi(t) \]

⇒ rectification: \( I_{\text{DC}} \neq 0 \)


Possible realization

\[ \begin{array}{c}
\times B \\
\end{array} \]

mesoscopic ring
Possible realization

mesoscopic ring

metallic strip subjected to potential difference
Possible realization

energy distribution in the strip

metallic strip subjected to potential difference

mesoscopic ring

\[ f_E \]
Perturbation theory

Non-equilibrium energy distribution function is introduced via ansatz:

\[ \delta G = -G^{(0)}R + G^{(0)}A, \quad h_E = 1 - 2f_E \]

\[ G^{(0)}_K = h_E \left( G^{(0)}_R - G^{(0)}_A \right), \quad h_E = 1 - 2f_E \]

Current\( = j \delta G_K \)
Perturbation theory

\[ \delta G = - \text{Hartree} + \text{Fock} \]

\[ \delta G = - \frac{\partial \Omega}{\partial \vec{A}} \iff \vec{j} = \frac{\partial \Omega}{\partial \vec{A}} \]

corrections to equilibrium current

Thermodynamic current:
Perturbation theory

\[ \delta G = - \delta G_{	ext{Hartree}} + \delta G_{	ext{Fock}} \]

\[ \delta G_K - h_E (\delta G_R - \delta G_A) \]

Thermodynamic current:

\[ \vec{j} = - \frac{\partial \Omega}{\partial \vec{A}} \Leftrightarrow \propto \frac{\partial}{\partial \vec{A}} \]

\(, \text{ etc.}\)
Before averaging over disorder

**Thermodynamic contribution:**

\[
I^{(eq)} = A_{RR} (U_R^\omega - 2U_R^0) (1 - h_E h_{E-\omega}) + \\
+ R_{AA} (U_A^\omega - 2U_A^0) (1 - h_E h_{E-\omega}) + (R_{RR} - A_{AA}) h_E U_K^\omega - \\
(1 - h_E h_{E-\omega})[R_{RR} (U_R^\omega - 2U_R^0) + A_{AA} (U_A^\omega - 2U_A^0)],
\]

**Kinetic contribution:**

\[
I^{(r)} = (A_{RA} - R_{RA})[ (h_E - h_{E-\omega}) U_K - \\
(1 - h_E h_{E-\omega}) (U_R - U_A)]
\]

If one substitutes \( \hat{j} \rightarrow \hat{1} \), \( \implies I^{(r)} \rightarrow 0 \)
\( \iff I^{(r)} \) is represented by off-diagonal matrix elements.
After the averaging over disorder

the effective interaction: \[
\frac{1}{2\nu_0 D_0 q^2} \int dE' R_\omega(E, E') \frac{(D_{0}\nu_E^2)^2 + \omega^2}{(D_{E'}q^2)^2 + \omega^2},
\]
\[\nu_E = \text{density of states, } D_E = v^2 \tau_0 \nu_0 / (3\nu_E) = \text{diffusion coeff.}
\]
\[R_\omega(E, E') = (h_E - h_{E-\omega})(1 - h_{E'}h_{E'-\omega}) - (h_{E'} - h_{E'-\omega})(1 - h_E h_{E-\omega})
\]
In equilibrium \[h_E = \tanh \frac{E}{2T}\] so that \[R_\omega(E, E') = 0.\]
Connection to the inelastic collision integral:

\[ St[E] = \int dE'd\omega P(\omega) R_\omega(E, E') \]

The global balance condition:

\[ \int dE St[E] = 0 \]

follows from

\[ \int dE dE' R_\omega(E, E') = 0 \]

\[ \iff \] for constant density of states one gets zero result.
Result of the calculations

Relaxation-induced (averaged) current:

\[ I^{(r)} = \sum_{n \geq 1} \sin \left[ 4\pi n \frac{\Phi}{\Phi_0} \right] I^{(r)}_n, \]

where

\[ I^{(r)}_n = -\frac{C_n e}{3h g} \int dE \left( \frac{\delta D_E}{D_0} \right) \left[ \tilde{T} \frac{\partial f_E}{\partial E} + f_E (1 - f_E) \right] \]

\[ g = \nu DS/L \] is the dimensionless conductance

\[ \tilde{T} = \int dE f_E (1 - f_E) \] is the effective temperature.
$D_E$ dependence from Kondo effect

Density of states

$$\nu_E$$

$$E$$

$$\tilde{T}$$

Diffusion coefficient

$$D_E$$

$$E$$

$$\nu_E D_E = \text{const}$$

Effective temperature $= \tilde{T} \sim V = \text{voltage on the strip}$
<table>
<thead>
<tr>
<th></th>
<th>Thermodynamical current</th>
<th>Non-equilibrium current</th>
</tr>
</thead>
<tbody>
<tr>
<td>amplitude</td>
<td>$0.1 \text{nA} \times \log^{-1} \left[ \frac{E_F}{E_T} \right]$</td>
<td>$100 \text{nA} \times \frac{1}{g} \times \frac{\delta D}{D}$,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{\delta D}{D} \sim \frac{n_K}{n_0} \times \frac{l}{\lambda_F}$</td>
</tr>
<tr>
<td>temperature dependence</td>
<td>$\exp \left[ -\frac{\tilde{T}}{E_T} \right]$</td>
<td>$\exp \left[ -\frac{1}{\sqrt{E_T \tau_\phi}} \right]$,</td>
</tr>
<tr>
<td></td>
<td>$E_T \sim 10^{-2} K$</td>
<td>$\frac{1}{\sqrt{\tau_\phi}} \propto \tilde{T}^{1/3}$</td>
</tr>
</tbody>
</table>
Conclusions

- In the presence of interaction and at a nonequilibrium electron energy distribution there is a kinetic contribution to the current (magnetization) related to relaxation.

- The disorder average of the relaxation-induced current is zero unless the diffusion coefficient is energy dependent.

- In contrast to the equilibrium persistent current, the relaxation-induced current is not exponentially small if the effective temperature is much larger than the Thouless energy.

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