

Magnetization of mesoscopic rings in a non-equilibrium steady state

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Journal reference:

O. L. Chalaev, V. E. Kravtsov, Phys. Rev. Lett., **89** 17 (176601).

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Thermodynamical vs dynamical equilibrium

Thermodynamical equilibrium:

$$\langle \hat{O} \rangle = \frac{1}{Z} \sum_{\mu} O_{\mu\mu} \exp[-\beta E_{\mu}]$$

Only diagonal matrix elements are relevant

Dynamical equilibrium:

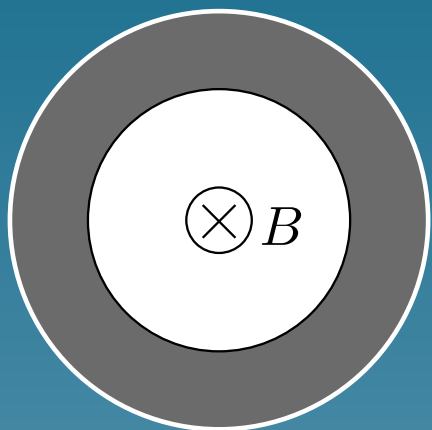
$$\begin{aligned} \langle \hat{O} \rangle &= \text{Tr} [\hat{\rho} \hat{O}] = \\ &= \sum_{\mu} \rho_{\mu\mu} O_{\mu\mu} + \sum_{\mu \neq \nu} \rho_{\nu\mu} O_{\mu\nu} \end{aligned}$$

Off-diagonal matrix elements are also relevant

- When is it relevant for a mesoscopic system?
- How to describe it by the diagrammatic technique?

Current in a mesoscopic ring with disorder: a bit of history

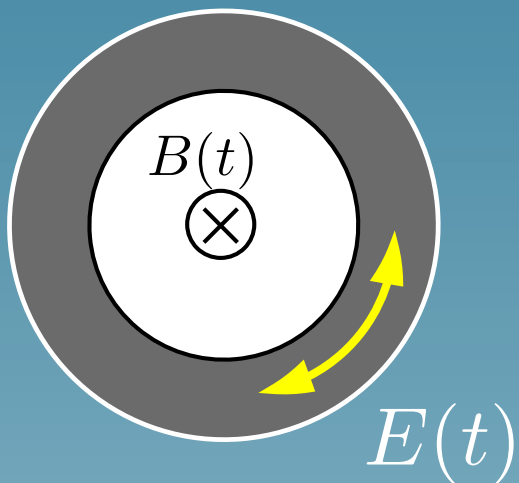
persistent current in equilibrium:



$$\Phi = \text{const}$$

V. Ambegaokar & U. Eckern, 1990.

Time-dependent external force:



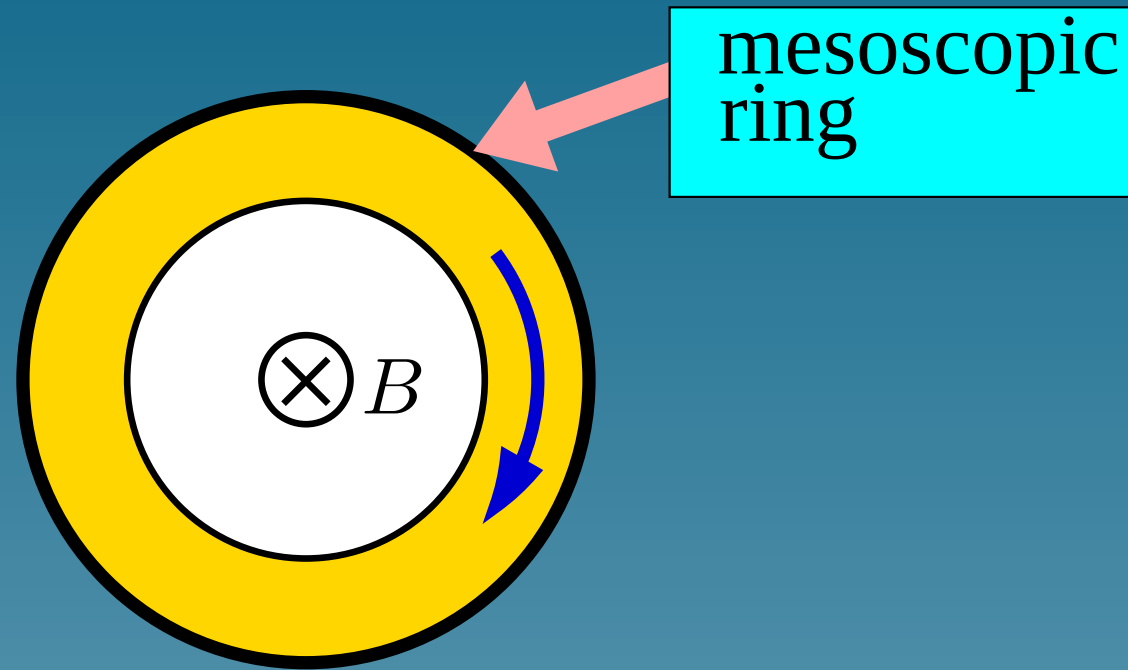
$$\Phi = \Phi_0 + \Phi(t)$$

$\Rightarrow$ rectification : $I_{\text{DC}} \neq 0$

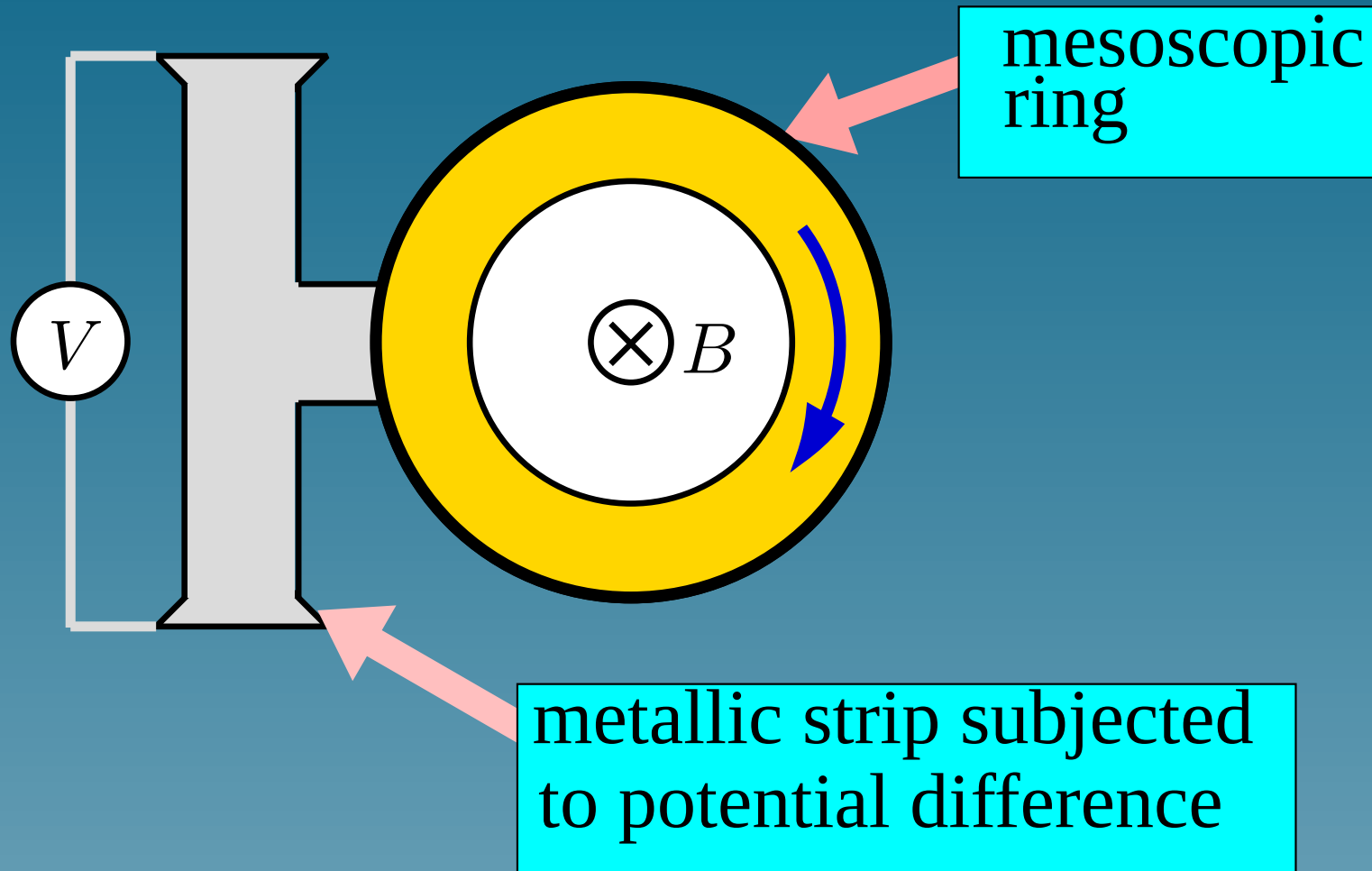
V. E. Kravtsov & B. L. Altshuler, 2000.

V. E. Kravtsov & V. I. Yudson, 1993.

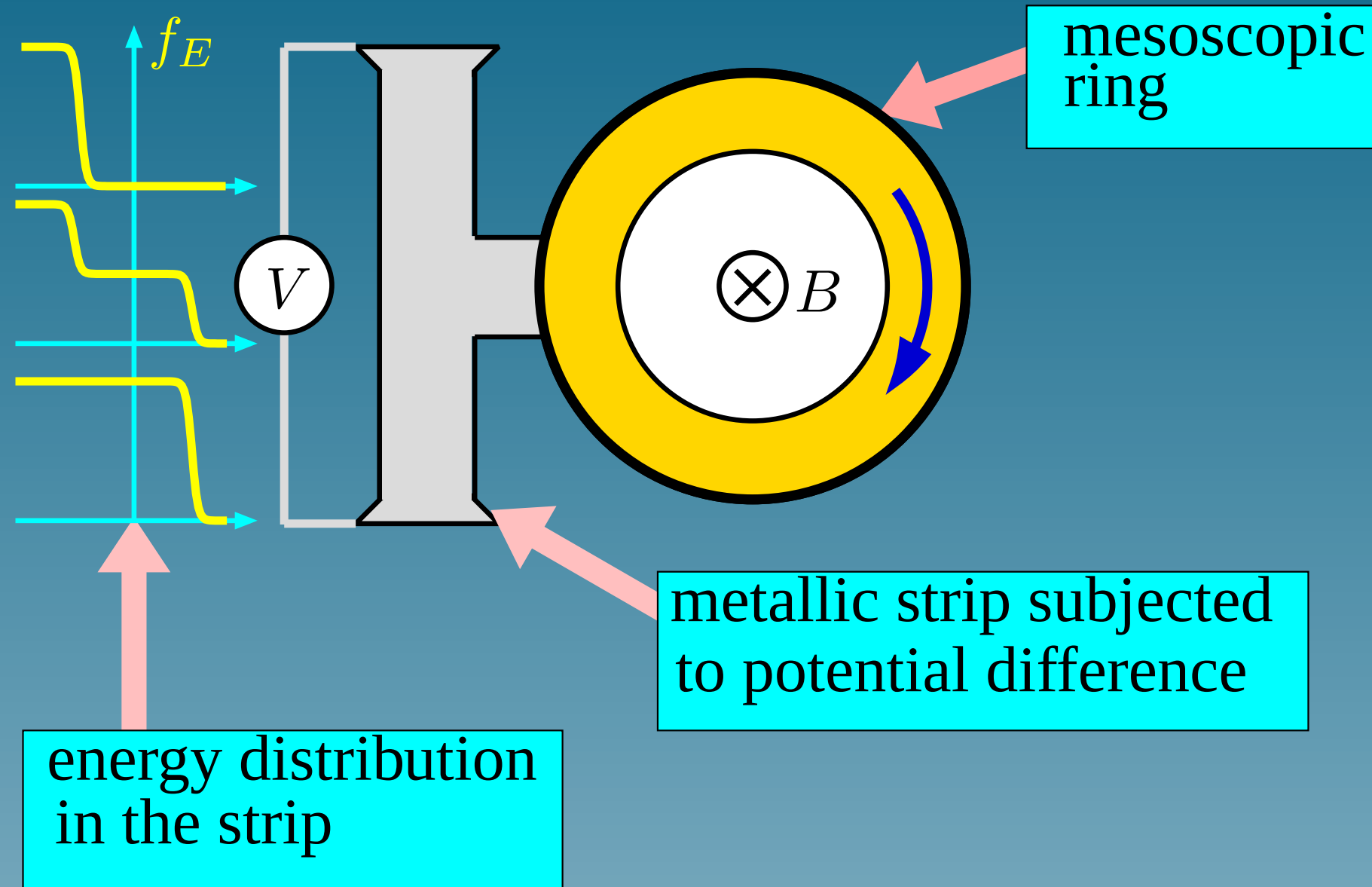
Possible realization



Possible realization



Possible realization



Perturbation theory

$$\delta G = - \text{Hartree} + \text{Fock}$$

Non-equilibrium energy distribution function is introduced via ansatz:

$$G_K^{(0)} = h_E \left(G_R^{(0)} - G_A^{(0)} \right), \quad h_E = 1 - 2f_E$$

$$\text{current} = \hat{j} G_K$$

Perturbation theory

$$\delta G = - \text{Hartree} + \text{Fock}$$

Diagram illustrating the perturbation theory expansion for δG . The expansion is shown as $\delta G = - \text{Hartree} + \text{Fock}$. The Hartree term is represented by a bubble diagram with a red horizontal line. The Fock term is represented by a bubble diagram with a red curved line. Below these, a dashed box labeled "corrections to equilibrium current" contains four diagrams: two for Hartree (labeled R, R and A, A) and two for Fock (labeled R, R and A, A). Arrows indicate the expansion of the Hartree and Fock terms into these correction diagrams.

Thermodynamic current:

$$\vec{j} = - \frac{\partial \Omega}{\partial \vec{A}} \Leftrightarrow \text{Fock diagram} \propto \frac{\partial}{\partial \vec{A}} \text{Hartree diagram}, \text{ etc.}$$

Diagram illustrating the thermodynamic current $\vec{j}$ as a derivative of the grand potential Ω with respect to the vector potential $\vec{A}$. The equation is $\vec{j} = - \frac{\partial \Omega}{\partial \vec{A}} \Leftrightarrow \text{Fock diagram} \propto \frac{\partial}{\partial \vec{A}} \text{Hartree diagram}, \text{ etc.}$. The diagrams are labeled with 'R' at the vertices.

Perturbation theory

$$\delta G = - \left[\text{Hartree} + \text{Fock} \right]$$

$\delta G_K - h_E (\delta G_R - \delta G_A)$

essentially non-equilibrium contribution

corrections to equilibrium current

Thermodynamic current:

$$\vec{j} = - \frac{\partial \Omega}{\partial \vec{A}} \Leftrightarrow \text{diagram} \propto \frac{\partial}{\partial \vec{A}} \text{diagram}, \text{ etc.}$$

Before averaging over disorder

Thermodynamic contribution:

$$I^{(eq)} = A\mathbf{RRR} (U_R^\omega - 2U_R^0) (1 - h_E h_{E-\omega}) + \\ + R\mathbf{AAA} (U_A^\omega - 2U_A^0) (1 - h_E h_{E-\omega}) + (R\mathbf{RRR} - A\mathbf{AAA}) h_E U_K^\omega - \\ - (1 - h_E h_{E-\omega}) [R\mathbf{RRR} (U_R^\omega - 2U_R^0) + A\mathbf{AAA} (U_A^\omega - 2U_A^0)],$$

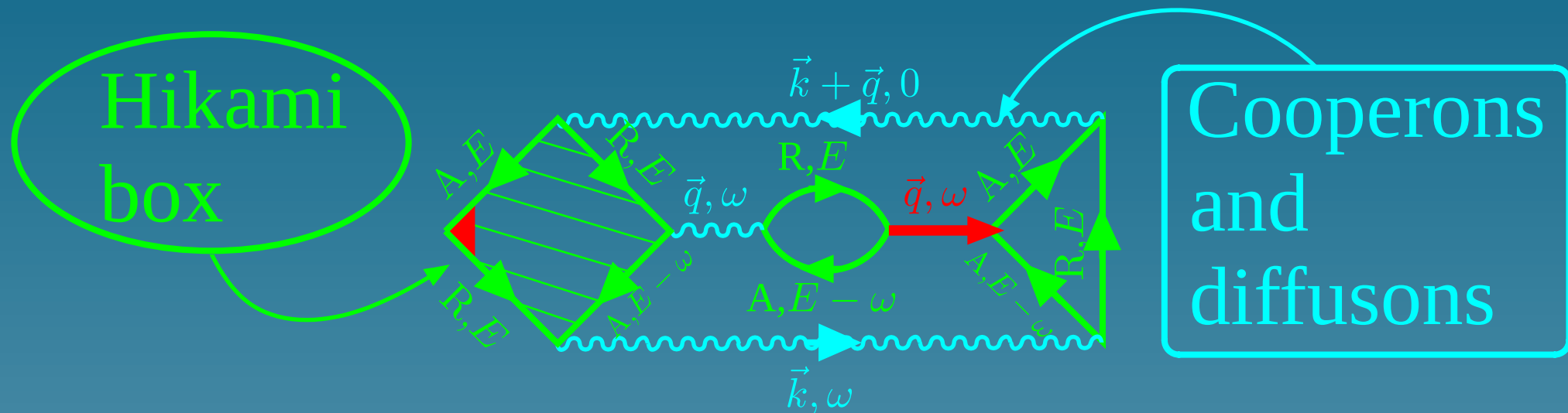
Kinetic contribution:

$$I^{(r)} = (A\mathbf{RAA} - R\mathbf{RAA}) [(h_E - h_{E-\omega}) U_K - \\ - (1 - h_E h_{E-\omega}) (U_R - U_A)]$$

If one substitutes $\hat{j} \rightarrow \hat{1}$, $\implies I^{(r)} \rightarrow 0$

$\Longleftarrow I^{(r)}$ is represented by off-diagonal matrix elements.

After the averaging over disorder



the effective interaction: $\frac{1}{2\nu_0 D_0 q^2} \int dE' R_\omega(E, E') \frac{(D_0 q^2)^2 + \omega^2}{(D_{E'} q^2)^2 + \omega^2},$

ν_E = density of states, $D_E = v^2 \tau_0 \nu_0 / (3\nu_E)$ = diffusion coeff.

$R_\omega(E, E') =$

$$(h_E - h_{E-\omega})(1 - h_{E'} h_{E'-\omega}) - (h_{E'} - h_{E'-\omega})(1 - h_E h_{E-\omega})$$

In equilibrium $h_E = \tanh \frac{E}{2T}$ so that $R_\omega(E, E') = 0$.

Connection to the inelastic collision integral:

$$St[E] = \int dE' d\omega P(\omega) R_\omega(E, E')$$

The global balance condition:

$$\int dE St[E] = 0$$

follows from

$$\int dE dE' R_\omega(E, E') = 0$$

$\iff$ for constant density of states one gets zero result.

Result of the calculations

Relaxation-induced (averaged) current:

$$I^{(r)} = \sum_{n \geq 1} \sin \left[4\pi n \frac{\Phi}{\Phi_0} \right] I_n^{(r)},$$

where

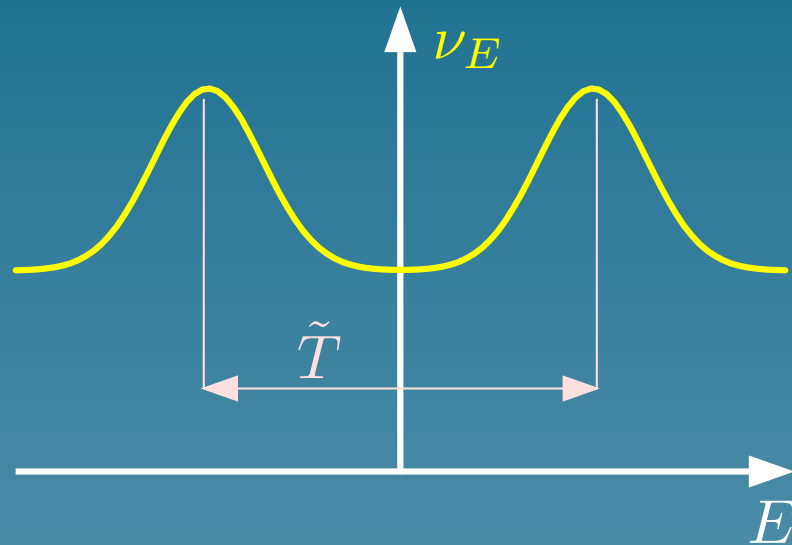
$$I_n^{(r)} = -\frac{C_n e}{3hg} \int dE \left(\frac{\delta D_E}{D_0} \right) \left[\tilde{T} \frac{\partial f_E}{\partial E} + f_E (1 - f_E) \right]$$

$g = \nu DS/L$ is the dimensionless conductance

$\tilde{T} = \int dE f_E (1 - f_E)$ is the effective temperature.

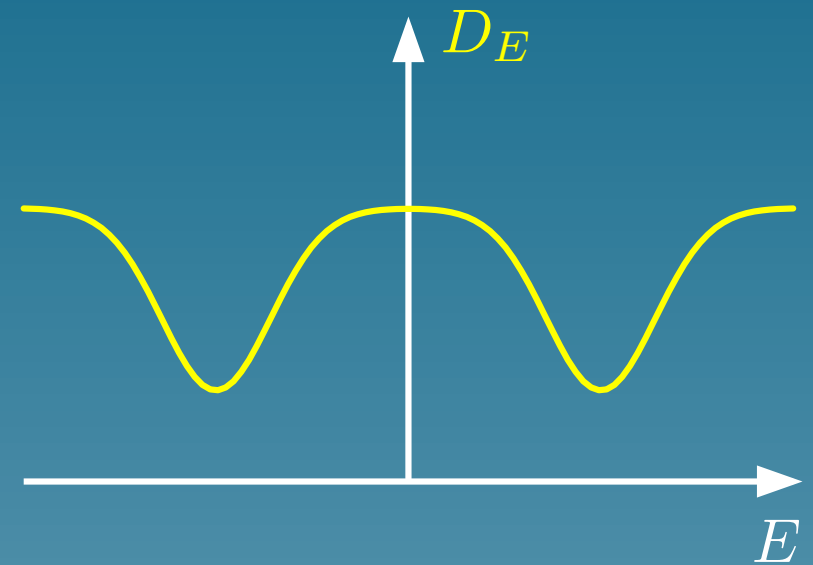
D_E dependence from Kondo effect

Density of states



$$\nu_E D_E = \text{const} \iff$$

Diffusion coefficient



effective temperature = $\tilde{T} \sim V$ = voltage on the strip

	Thermodynamical current	Non-equilibrium current
amplitude	$0.1\text{nA} \times \log^{-1} \left[\frac{E_F}{E_T} \right]$	$100\text{nA} \times \frac{1}{g} \times \frac{\delta D}{D},$ $\frac{\delta D}{D} \sim \frac{n_K}{n_0} \times \frac{l}{\lambda_F}$
temperature dependence	$\exp \left[-\frac{\tilde{T}}{E_T} \right],$ $E_T \sim 10^{-2} K$	$\exp \left[-\frac{1}{\sqrt{E_T \tau_\phi}} \right],$ $\frac{1}{\sqrt{\tau_\phi}} \propto \tilde{T}^{1/3}$

Conclusions

- In the presence of interaction and at a nonequilibrium electron energy distribution there is a kinetic contribution to the current (magnetization) related to relaxation.
- The disorder average of the relaxation-induced current is zero unless the diffusion coefficient is energy dependent.
- In contrast to the equilibrium persistent current, the relaxation-induced current is not exponentially small if the effective temperature is much larger than the Thouless energy.

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