# Anisotropic conductivity of disordered 2DEGs due to spin-orbit interactions

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## The Hamiltonian: spin-orbit interaction and disorder

2D electron gas in random (disorder) potential + Rashba & Dresselhaus SOI:

$$\hat{H} = \frac{\hat{p}^2}{2m} + V_s + U(\vec{r}), \quad \overline{U(\vec{r})U(\vec{r}\,')} = (m\tau)^{-1}\delta(\vec{r}-\vec{r}\,'),$$
$$V_s = a(\sigma_1\hat{p}_y - \sigma_2\hat{p}_x) + b(\sigma_1\hat{p}_x - \sigma_2\hat{p}_y).$$

Anisotropic spectrum already without disorder:

$$E = \frac{p^2}{2m} \pm \frac{\Delta_{\vec{p}}}{2}, \quad \Delta_{\vec{p}} = 2\sqrt{p_x^2(a+b)^2 + p_y^2(a-b)^2}.$$

Does it result in anisotropic conductivity?

## Kubo-Greenwood formula for conductivity

$$\sigma_{\alpha\beta} = \frac{e^2}{h}\mathrm{Tr}\left[v_\alpha \hat{G}_{\mathrm{R}} v_\beta \hat{G}_{\mathrm{A}}\right], \quad \alpha,\beta = x,y,$$
$$v_\alpha = \frac{i}{\hbar}[\hat{H}, r_\alpha], \quad \hat{G}_{\mathrm{R}/A} = [E_{\mathrm{F}} - \hat{H} \pm i0]^{-1}.$$

## Expansion parameters

From Ref. [2]:

$$\sigma_{yy}(-a,b) = \sigma_{xx}(a,b) = \sigma_{xx}(-a,-b).$$

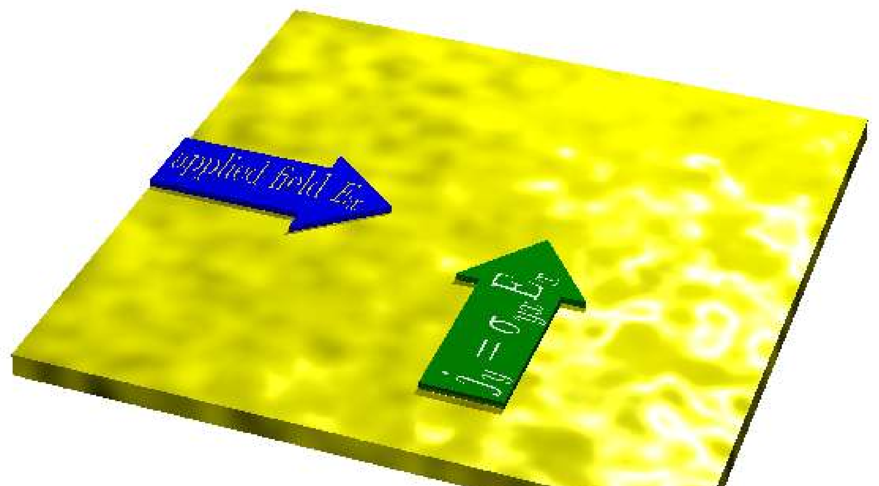
Anisotropic part of the conductivity tensor in 2D:

$$\delta\sigma - \sigma_0\,\mathrm{Tr}\left[\frac{\delta\sigma}{2}\right] = \frac{e^2}{h}\sigma_3 \sum_{m,n,r\geq 0} S^r_{mn}\frac{x^m\delta^{2n+1}}{(p_{\mathrm{F}}l)^r},$$

where SOI amplitude $x = 2p_{\mathrm{F}}\tau\sqrt{a^2+b^2} \ll 1$,

spectrum anisotropy $\delta = \dfrac{2ab}{a^2+b^2} \ll 1$.

# 2D case



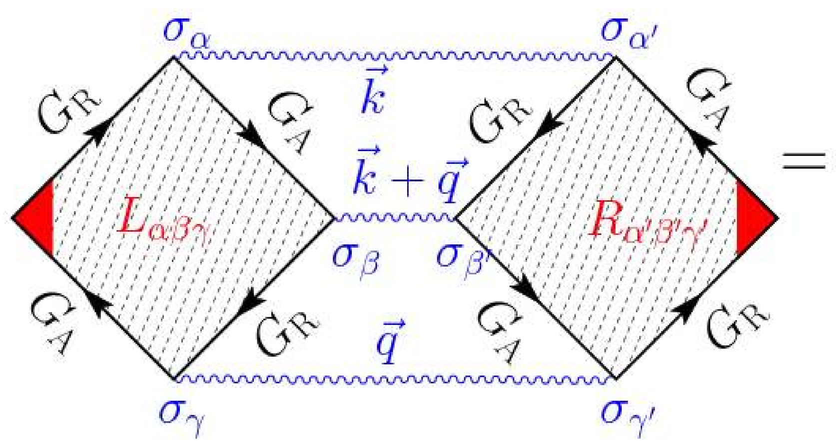

### Zero frequency $\omega = 0$

Non-analytical result:

$$\sigma_{xy} = -5.6\times 10^{-3}\frac{2ab}{a^2+b^2}\frac{e^2}{h}\frac{1}{p_{\mathrm{F}}l}.$$

For vanishing SOI $\sigma_{xy}$ can be finite!

Expression for one diagram (out of three most relevant ones):




$$= \int\frac{\mathrm{d}^2k}{(2\pi)^2}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\sum_{\alpha,\beta,\gamma=0}^{3}\sum_{\alpha',\beta',\gamma'=0}^{3} L_{\alpha\beta\gamma}D^{\alpha\alpha'}_{\vec{k}}D^{\beta'\beta}_{\vec{k}+\vec{q}}D^{\gamma\gamma'}_{\vec{q}}R_{\alpha'\beta'\gamma'}$$

–HUGE expression, can not be treated without computer.

### Finite frequency $\omega \neq 0$

The non-analiticity is removed:

$$\sigma_{xy} = -2\cdot 0.25\cdot\frac{-2i\omega\tau\cdot 2x_a x_b}{(x_a^2+x_b^2-2i\omega\tau)^2}\frac{e^2}{2\pi}\frac{1}{p_{\mathrm{F}}l},$$

$$2x_a x_b \ll x_a^2 + x_b^2 \ll \omega\tau \ll 1, \quad x_a = 2p_{\mathrm{F}}a\tau, \; x_b = 2p_{\mathrm{F}}b\tau.$$

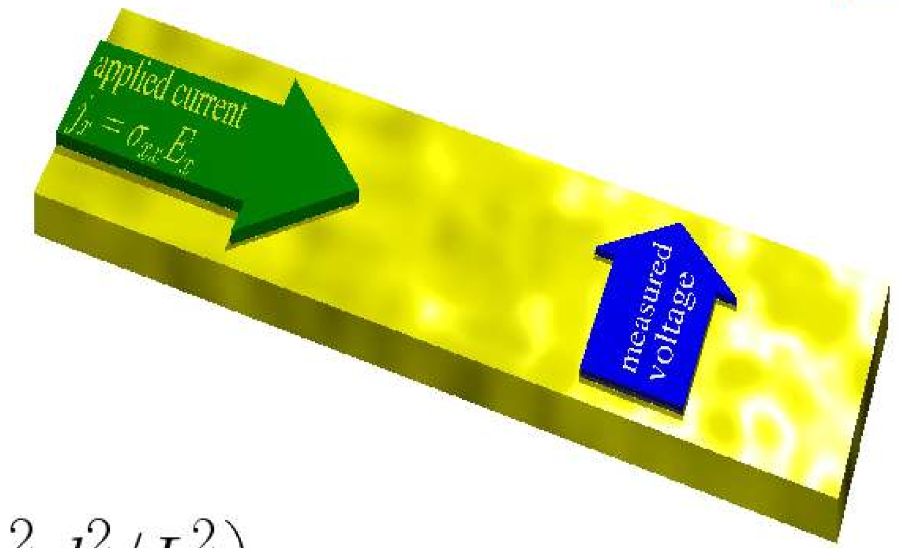
Interpretation – dephasing:

$$\sigma_{xy} = \begin{cases} 5.6\times 10^{-3}\cdot\dfrac{\tau_- - \tau_+}{\tau_- + \tau_+}\dfrac{e^2}{2\pi}\dfrac{1}{E_{\mathrm{F}}\tau}, & \tau_\pm \ll \tau_\phi, \\ 0.13\cdot\left(\dfrac{\tau_\phi}{\tau_+} - \dfrac{\tau_\phi}{\tau_-}\right)\dfrac{e^2}{2\pi}\dfrac{1}{E_{\mathrm{F}}\tau}, & \tau_\phi \ll \tau_\pm, \end{cases}$$

where [1] $2\tau/\tau_\pm = (x_a \mp x_b)^2$.

# Quasi–1D case

### Long wire




Suppose

$$L_\perp \gg l$$

but

$$E_{c\perp}\tau/\hbar \gg \tilde{x}^2 \equiv \max\left(x^2, l^2/L_\phi^2\right),$$

where $E_{c\perp} = D/L_\perp^2$ and $D = lv_{\mathrm{F}}/2$.

The diffusons and cooperons become one-dimensional:

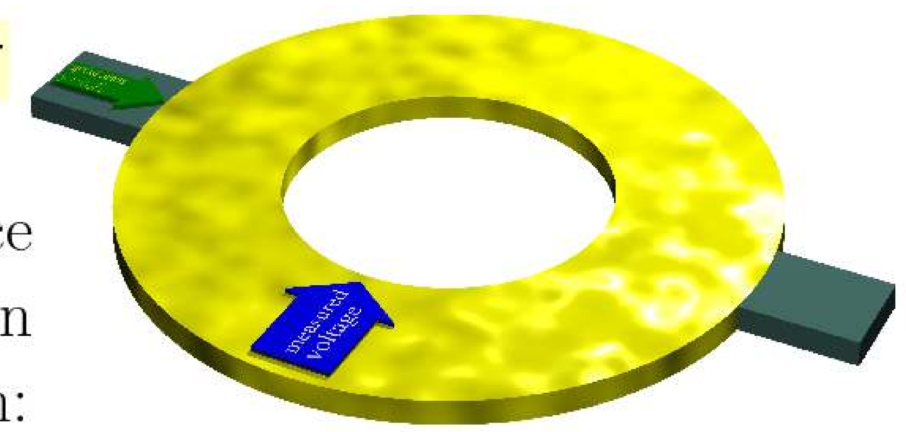
$$\int\frac{\mathrm{d}^2k}{(2\pi)^2}\int\frac{\mathrm{d}^2q}{(2\pi)^2} \to \frac{1}{L_\perp^2}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{\mathrm{d}q_x}{(2\pi)^2},$$

$\Longrightarrow$ the effect becomes $l^2/(\tilde{x}L_\perp)^2 \gg 1$ times larger:

in quasi – 1D for $x^2 \gg l/L_\phi$ (in the rotated coordinate system)

$$\delta\sigma = \frac{e^2}{h}\frac{\hbar}{p_{\mathrm{F}}l}\frac{l^2}{x^2L_\perp^2}\left[\begin{pmatrix}-0.39 & 0\\ 0 & 6.7\end{pmatrix} + \delta\begin{pmatrix}-852 & 0\\ 0 & 13\end{pmatrix}\right].$$

### Ring pierced by magnetic flux




Time-reversal invariance is broken $\Longrightarrow$ cooperon differs from the diffuson:

$$C^{\alpha\beta}_{\vec{q}} = D^{\alpha\beta}_{\vec{q}-2e\vec{A}/c}.$$

$\Longrightarrow$ small-momenta divergences are uncompensated!

$$\sigma_{xy} = \frac{e^2}{h}\left[\cos\left(2\pi\frac{\phi}{\phi_0}\right) - 1\right]\frac{lL_\phi}{\tilde{x}L_\perp^2}\frac{\hbar}{p_{\mathrm{F}}l}(\Sigma_0 + \Sigma_1\delta),$$

$$\Sigma_i = a_i e^{-xL/l} + \exp\left[-\frac{xL}{2l}\sqrt{2\sqrt{2}-1}\right]\times$$
$$\times\left\{b_i\cos\left[\frac{xL}{2l}\sqrt{2\sqrt{2}+1}\right] + c_i\sin\left[\frac{xL}{2l}\sqrt{2\sqrt{2}+1}\right]\right\}.$$

## Diagrams are calculated automatically

This calculation is impossible without the usage of computer algebra software [3]. We developed a program that

- Generates all relevant diagrams up to the given number of loops.
- Generates and automatically calculates all Hikami boxes.
- Performs integration over the cooperon/diffuson momenta. (This last step is specific for different problems ; requires human internvention.)

See [4] and `http://shalaev.pochta.ru/work/diagrams.html`

**References**

[1] Averkiev, N. S., Golub, L. E., and Willander, M. (2002) *J. Phys.: Condens. Matter* **14**, R271.

[2] Chalaev, O. and Loss, D. (2008) *Phys. Rev. B* **77**, 115352.

[3] http://maxima.sourceforge.net/.

[4] Chalaev, O. and Loss, D. Spin orbit-induced anisotropic conductivity of a disordered 2DEG `cond-mat/0902.3277`.