Anisotropic conductivity of disordered 2DEGs due to spin-orbit interactions

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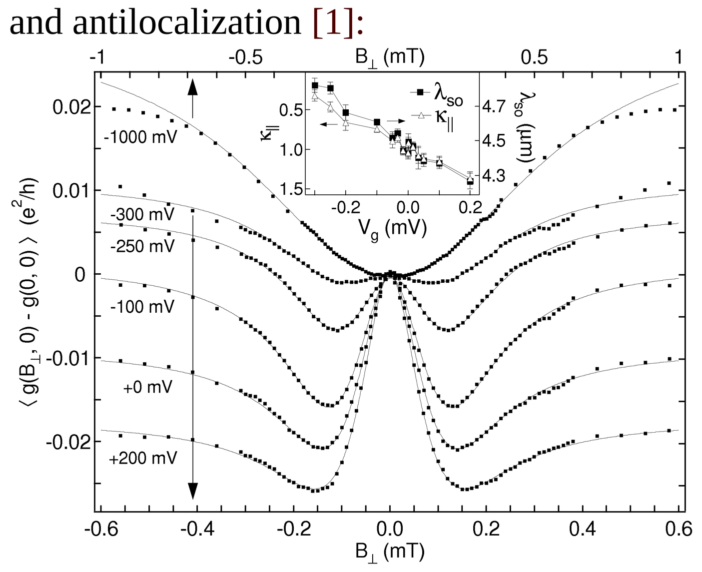
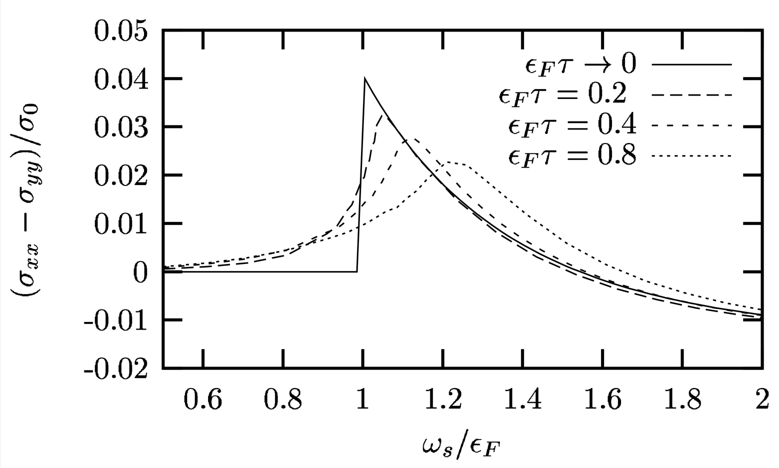
SOI = spin-orbit interaction

1 Introduction: previously studied influence of the spin-orbit on conductivity

Crossover between weak localization and antilocalization [1]:



Anisotropic magnetoconductivity in the presence of Rashba SOI [2]:



Our motivation:
Is there an anisotropic conductivity without magnetic field?

2 The Hamiltonian: spin-orbit interaction and disorder

2D electron gas in random (disorder) potential + Rashba & Dresselhaus SOI:

$$\hat H = \frac{\hat p^2}{2m} + V_s + U(\mathbf r), \quad V_s = a(\sigma_1\hat p_y - \sigma_2\hat p_x) + b(\sigma_1\hat p_x - \sigma_2\hat p_y).$$

Randomly placed impurities create disorder potential (the overline denotes averaging over impurities configurations):

$$\overline{U(\mathbf r)U(\mathbf r')} = (m\tau)^{-1}\delta(\mathbf r - \mathbf r').$$

Relative probability of a particular disorder realization:

$$P[U] \sim \exp\left[-\frac{m\tau}{2}\int U^2(\mathbf r)\mathrm d\mathbf r\right].$$

Averaged (over different disorder realizations) conductivity:

$$\sigma_{\alpha\beta} = \frac{e^2}{h}\mathrm{Tr}\left[v_\alpha \hat G_{\mathrm R} v_\beta \hat G_{\mathrm A}\right], \quad \alpha,\beta = x,y,$$
$$v_\alpha = i[\hat H, r_\alpha], \quad \hat G_{\mathrm R/A} = [E_{\mathrm F} - \hat H \pm i0]^{-1}.$$

3 Anisotropic energy spectrum due to spin-orbit

Raimondi & Schwab'02 [2]: Rashba SOI + Zeeman magnetic field:

$$H = \frac{\hat p^2}{2m} + a(\sigma_1\hat p_y - \sigma_2\hat p_x) - \frac{\omega_B}{B}\sigma\mathbf B.$$

Anisotropic spectrum:

$$E = \frac{p^2}{2m} \pm \frac{\Delta_{\mathbf p}}{2}, \quad \Delta_{\mathbf p} = 2\sqrt{(ap_y - \omega_B)^2 + a^2p_x^2}.$$

– leads to anisotropic conductivity tensor.

Our system:

$$H = \frac{\hat p^2}{2m} + a(\sigma_1\hat p_y - \sigma_2\hat p_x) + b(\sigma_1\hat p_x - \sigma_2\hat p_y).$$

Anisotropic spectrum:

$$E = \frac{p^2}{2m} \pm \frac{\Delta_{\mathbf p}}{2}, \quad \Delta_{\mathbf p} = 2\sqrt{p_x^2(a+b)^2 + p_y^2(a-b)^2}.$$

Does it result in anisotropic conductivity?

4 Symmetry properties and expansion parameters

Symmetries:

- $a = \pm b$: $s_x \pm s_y$ becomes conserved quantity, and conductivity becomes SOI-independent: $\sigma_{\alpha\beta}(a = \pm b) = \sigma_{\alpha\beta}(a = b = 0)$.
- σ_{xy} changes sign when ab changes sign.

Expansion parameters:

- SOI amplitude $x = 2p_{\mathrm F}\tau\sqrt{a^2+b^2}$.
- Spectrum anisotropy $\delta = \frac{2ab}{a^2+b^2}$.

$$\sigma_{xy} = \frac{e^2}{h}\sum_{m,n\ge 0} S_{mn}x^m\delta^{2n+1}.$$

We calculate S_{00} using disorder averaging diagrammatic technique.

5 Cooperons and diffusons in the presence of spin-orbit

$$D^{\alpha\beta}(\mathbf q) = \frac{1}{4\pi\nu\tau}[E_4 - X_D(\mathbf q)]^{-1}_{\alpha\beta}, \quad C^{\alpha\beta}(\mathbf q) = \frac{1}{4\pi\nu\tau}[E_4 - X_C(\mathbf q)]^{-1}_{\alpha\beta},$$

where

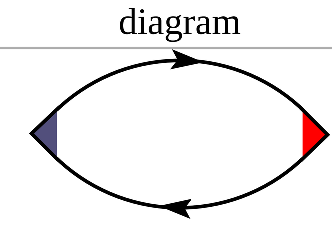
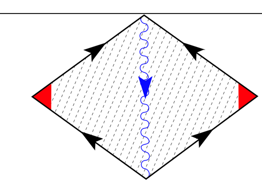
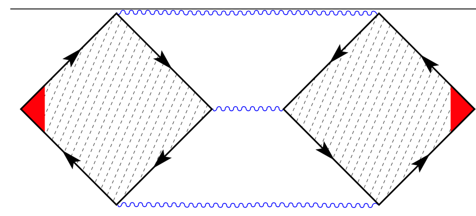
$$X_D^{\alpha\beta}(\mathbf q) = X_C^{\alpha\beta}(\mathbf q) = \frac{1}{4\pi\nu\tau}\int\frac{\mathrm d^2p}{(2\pi)^2}\mathrm{Tr}[\tilde\sigma_\alpha G_{\mathrm R}^E(\mathbf p)\tilde\sigma_\beta G_{\mathrm A}^{E-\omega}(\mathbf p - \mathbf q)],$$

and

$$\tilde\sigma_0 = \sigma_0, \quad \tilde\sigma_{1,2} = \frac{\sigma_2 \pm \sigma_1}{\sqrt 2}, \quad \tilde\sigma_3 = \sigma_3.$$

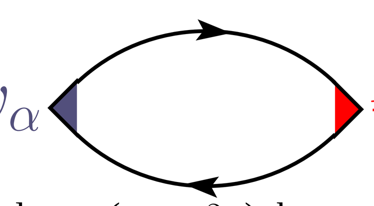
6 Contributions to conductivity – the loop expansion

We assume that $p_{\mathrm F}l \gg 1$, where l is the mean free path of electrons.
The contribution of a diagram can be estimated based on the number of loops:

diagram	number of loops	relative smallness
	0 (ZLA)	1
	1 (WL)	$(p_{\mathrm F}l)^{-1}$
	2	$(p_{\mathrm F}l)^{-2}$

Every loop brings smallness $\sim (p_{\mathrm F}l)^{-1} \ll 1$.

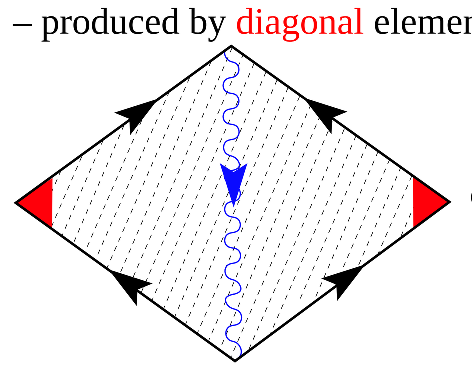
7 Zero Loop approximation (ZLA)

v_α  $\frac{p_\beta}{m} = \frac{e^2}{h}\frac{p_{\mathrm F}l}{2}\left[1 + \frac{a^2+b^2}{v_{\mathrm F}^2}\right]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

– almost (up to 2π) the same as obtained using Boltzmann kinetic equation [3].
This result can not be complete since for $a = \pm b$ the conductivity must be SOI-independent.

8 Weak localization

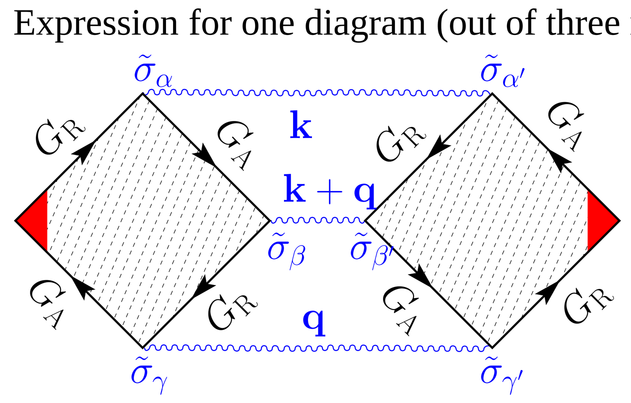
– produced by diagonal elements of the cooperon matrix:

 $\propto C^{00} - C^{11} - C^{22} - C^{33}.$

– isotropic (within the considered accuracy).

9 Calculation procedure for diagrams with two loops

Expression for one diagram (out of three most relevant ones):



$$= \int\frac{\mathrm d^2k}{(2\pi)^2}\int\frac{\mathrm d^2q}{(2\pi)^2}\sum_{\alpha,\beta,\gamma=0}^{3}\sum_{\alpha',\beta',\gamma'=0}^{3} L_{\alpha\beta\gamma}D^{\alpha\alpha'}_{\mathbf k}D^{\beta\beta'}_{\mathbf k+\mathbf q}D^{\gamma\gamma'}_{\mathbf q}R_{\alpha'\beta'\gamma'}$$

–HUGE expression, can not be treated without computer.

$$\int\frac{\mathrm d^2k}{(2\pi)^2}\int\frac{\mathrm d^2q}{(2\pi)^2} = \int_0^\infty\frac{\mathrm dk k}{2\pi}\int_0^\infty\frac{\mathrm dq q}{2\pi}\int_0^{2\pi}\frac{\mathrm d\phi}{2\pi}\int_0^{2\pi}\frac{\mathrm d\psi}{2\pi}$$

– done analytically and numerically.

10 Non-analytical SOI-dependence when $L_\phi = \infty$

At zero frequency:

$$\sigma_{xy} = -5.6\times 10^{-3}\frac{2ab}{a^2+b^2}\frac{e^2}{h}\frac{1}{p_{\mathrm F}l}.$$

For vanishing SOI σ_{xy} can be finite!
A similar non-analyticity also occurs in WL (e.g., [4, 1])
At large frequency:

$$\sigma_{xy} = -0.25\cdot\frac{-2i\omega\tau\cdot 2x_ax_b}{(x_a^2 + x_b^2 - 2i\omega\tau)^2}\frac{e^2}{h}\frac{1}{p_{\mathrm F}l},$$

where $x_a = 2p_{\mathrm F}a\tau$, $x_b = 2p_{\mathrm F}b\tau$, $2x_ax_b \ll x_a^2 + x_b^2 \ll \omega\tau \ll 1$.

11 Connection with dephasing

Let us take the dephasing into account by substituting $-i\omega\tau \longrightarrow \tau/\tau_\phi$.
$\Longrightarrow$ The analyticity is restored due to the dephasing

$$\delta\sigma = \begin{cases} 5.6\times 10^{-3}\cdot\frac{\tau_- - \tau_+}{\tau_- + \tau_+}\frac{e^2}{h}\frac{1}{E_{\mathrm F}\tau}, & \tau_\pm \ll \tau_\phi,\\ 0.13\cdot\left(\frac{\tau_\phi}{\tau_+} - \frac{\tau_\phi}{\tau_-}\right)\frac{e^2}{h}\frac{1}{E_{\mathrm F}\tau}, & \tau_\phi \ll \tau_\pm,\end{cases}$$

where spin-relaxation times $\tau_\pm$ are [5]

$$2\tau/\tau_\pm = (x_a \mp x_b)^2.$$

12 Summary

- The conductivity tensor becomes anisotropic in the presence of both Rashba and Dresselhaus SOI.
- This anisotropy has been missed within the Boltzmann equation approach.
- The dependence of $\delta\sigma$ on the SOI amplitude is singular for $\omega = 0$ and $L_\phi = \infty$.
- Manipulating diagrams and their expressions has to be performed on computer. (Mixed analytical-numerical treatment.)

13 Further research

Diagrams contain divergences, which mutually cancel in a system with time-reversal [6].
Will there be large anisotropic magnetoresistance?
See [7] for more details.
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References

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