Theory of Spin Hall conductivity in n-doped GaAs

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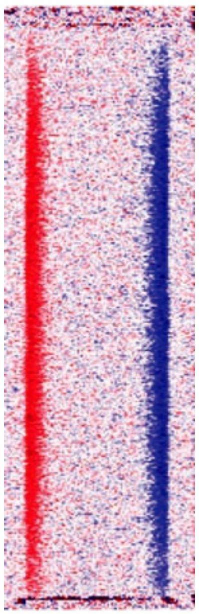
Experimental measurement of extrinsic spin-current

See Y. K. Kato et al., Science 2004:
Measureing spin accumulation with Kerr microscope (magnetooptical Kerr effect)



sample size: $77\mu\mathrm{m} \times 300\mu\mathrm{m}$,
$T = 30\mathrm{K}$,
laser beam radius$= 1.1\mu\mathrm{m}$,
electric field $E = 10\mathrm{m}V/\mu m$.

Hamiltonian with *extrinsic* spin-orbit interaction:

$$H = \frac{\hbar^2 k^2}{2m^*} + V(\vec{r}) + \lambda \boldsymbol{\sigma} \cdot \left(\vec{k} \times \nabla V \right),$$

Plan

- Spin-generalization for the kinetic equation of scattering off impurities.
- Cunning ansatz for the distribution function.
- Side jump: Cunning connection of SHE with anomalous Hall effect.
- Estimate of SO amplitude (also cunning).
- Conclusion.

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Kinetic equation

The interaction is neglected, so kinetic equation must be a spin generalization of ([1]2.80): STOP I forgot $\vec{E}$, see ([1]3.28).

$$\left[\partial_T + \vec{v}\vec{\nabla}_{\vec{R}} - \vec{\nabla}_{\vec{R}} U \vec{\nabla}_{\vec{k}}\right] \hat{f}(\vec{k}) = n_i \sum_{\vec{k}';\; k'=k} \frac{\hbar k}{m^*} \frac{d\overleftrightarrow{\sigma}}{d\Omega} \left[\hat{f}(\vec{k}) - \hat{f}(\vec{k}')\right]$$

Only $\propto \vec{\nabla} U$ term is relevant on the l.h.s. Parametrization:

$$\hat{f} = \left[f_0(\vec{k}) + \phi(\vec{k})\right] \mathbb{1} + \vec{f}(\vec{k}) \cdot \vec{\sigma},\quad f_0(\vec{k}) = \text{eq. distr. function},$$

$$\frac{d\overleftrightarrow{\sigma}}{d\Omega} \left[\hat{f}(\vec{k}) - \hat{f}(\vec{k}')\right] = I(\vartheta) \left[\hat{f}(\vec{k}) - \hat{f}(\vec{k}')\right] + I(\vartheta) S(\vartheta) \vec{\sigma} \cdot \vec{n} \left[\phi(\vec{k}) - \phi(\vec{k}')\right].$$

$I(\vartheta) =$ spin indep. coef., $S(\vartheta) =$ Sherman func.

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Cunning ansatz

Spin-independent term is set to $\phi(\vec{k}) = \vec{k} \cdot \vec{E}\, C_k$. In this I believe, but why it is "justified by Mott scattering theory" that

$$\vec{f}(\vec{k}) = \left(\vec{E} \times \vec{k}\right) D_k ?$$

Why terms $\propto \vec{E}$ and $\propto \vec{k}(\vec{k}\vec{E})$ are not allowed for $\vec{f}(\vec{k})$?

The resulting distribution function: STOP see ([1]3.36)

$$\hat{f}(\vec{k}) = f_0(k) + \vec{k} \cdot \left[\vec{E} + \frac{\gamma_k}{2}\left(\vec{\sigma} \times \vec{E}\right)\right] C_k$$

$$j^{\mu}_{\mathrm{SS},\,\kappa} = \mathrm{Tr}\,\sigma_\mu \int \frac{d^3k}{(2\pi)^3} \frac{\hbar\, k_\kappa}{m^*} \hat{f}(\vec{k}) = \frac{\gamma}{2e} \varepsilon^{\kappa\mu\nu} \left(\vec{J}_0\right)_\nu ,$$

where $\vec{J}_0$ is the charge current without SO.

Side jump

Cunning trick:
The SO amplitude "Because λ is small and spin relaxation is of order $\lambda^2 \ldots$" let us say that

$$\vec{j}^{\,\mu}_{\mathrm{SH}} = e^{-1}\left(\vec{J}^{\uparrow}_{\mathrm{AH}} - \vec{J}^{\downarrow}_{\mathrm{AH}}\right),$$

and just use the results for charge current calculated by other people long-long ago.

$$j^{\mu}_{\mathrm{SJ},\,\kappa} = -2n\lambda\frac{e}{\hbar}\,\varepsilon^{\kappa\mu\nu}E_{\nu}.$$

The sum $\vec{j}_{\mathrm{SS}} + \vec{j}_{\mathrm{SJ}}$ provides the total spin-Hall current.

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During the APCTP workshop E. Мищенко told me that it must be possible to get the effect of side jump from the kinetic equation, only one has to understand, what is the important set of diagrams. E.g., weak localization one catches with the kinetic equation on p. [1]344.

Estimates

Before we wanted to use the results which were obtained (long ago) for scattering off lonely atoms.

$$V = -e^{-q_s r}\, e^2/\epsilon r$$

So let us plug into those results non-standart atomic parameters: atomic number $Z = 1/\epsilon$, (for GaAs) $\epsilon = 12.4$ and $m^* = 0.0665\, m_0$, thus $c^* \approx c/79$ and $\alpha^* Z \approx 1/21$, with fine structure constant $\alpha^* = e^2/\hbar c^*$.

– in this way we get SO amplitude $\lambda = \hbar^2/4\,(m^* c^*)^2$ and $S(\vartheta)$, from where we get skewness.

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Results and conclusions

- There are 2 contributions to extrinsic SHE: skew scattering and side jump.
- Comparison with experiment: $+40\%$ – within error bars.

this document is available on http://shalaev.pochta.ru and here.

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Help desk: Anomalous Hall effect

– the contribution to the Hall field comming from magnetization of a ferromagnet, which corresponds to the second term in

$$E_y = RB_z j_x + R_s 4\pi M_z j_x.$$

Comes from spin-orbit interaction, which (in a ferromagnet) is proportional to magnetization and creates asymmetry in electron scattering. Finally, there is "adiabatic Hall effect", see §6.2 on p. [2]97.

See the theory in [3].

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Help desk: Thomas-Fermi model (of screening)

click here for more. – quasiclassical static model of an atom, used for atoms with many electrons. The electron density is assymed to be spherically-symmetric. Electrons are considered, as a Fermi liquid with a Fermi momentum $p_{\mathrm{F}}(\vec{r})$:

$$\epsilon(\vec{p}, r) = \frac{p^2}{2m} + e\phi(\vec{r}), \quad \Delta\phi(\vec{r}) = 4\pi\rho(\vec{r}), \quad \epsilon(p_{\mathrm{F}}, \vec{r}) = 0$$

From here one writes an equation for the scalar potential $\phi(r)$, which is solved numerically. The solution shows, how the attractive potential of the kernel is screened by the electron cloud.

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Help desk: ур-ние Больцмана в классической механике

$$\frac{\partial f}{\partial t} = -\vec{v}\vec{\nabla} f + \mathrm{St}\, f,$$

где

$$\mathrm{St}\, f = \int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma' \mathrm{d}\Gamma_1' w(\Gamma, \Gamma_1; \Gamma', \Gamma_1')(f' f_1' - f f_1),$$

где $f \equiv f(t, \vec{r}, \Gamma)$ – функция распределения, Γ – точка фазового пространства (то есть пространства координат и импульсов, полностью характеризующее систему), за исключением координат частиц газа (стр. [4]15). Таким образом, для одноатомного газа координаты не входят в Γ. (Если же речь идёт о молекулярном газе, в Γ входят относительные координаты атомов молекулы.)

А ещё Ландау очень лихо вкручивает (без объяснения) сечение рассеяния в интеграл столкновений:

$$\mathrm{St}\, f = \int v_{rel}(f' f_1' - f f_1) \mathrm{d}\sigma \mathrm{d}^3 p_1. \qquad ([4]3.9)$$

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Help desk: Mott scattering

Scattering due to the interaction of a spin of an electron with its orbital momentum in the electric field of a scatterer. Assymetric with respect to the plane containing spin and momentum of incoming electrons. Initially unpolarized electrons become polarized, $\vec{P} = S(\theta)\vec{h}$, where $\vec{h}$ is a unit vector $\perp$ to the scattering plane $(\vec{p}, \vec{p}\,')$; $S(\theta)$ is a Sherman function (which has a non-trivial angular dependence)

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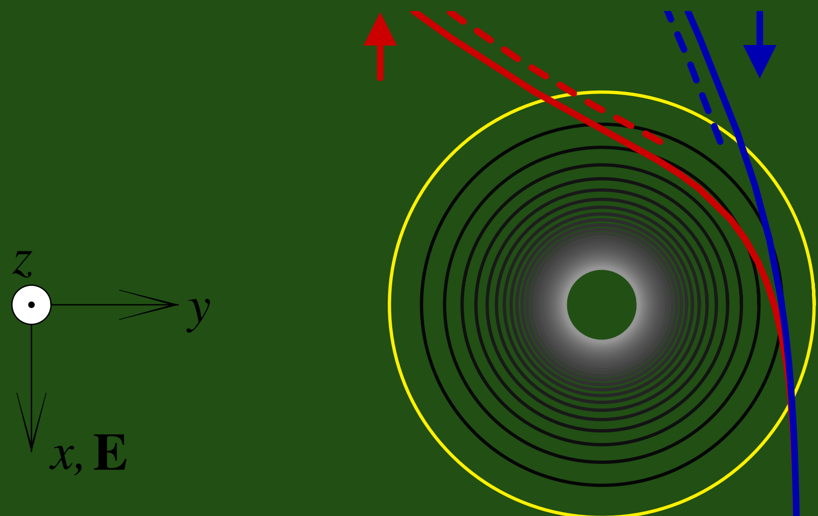
Help desk: Side jump effect

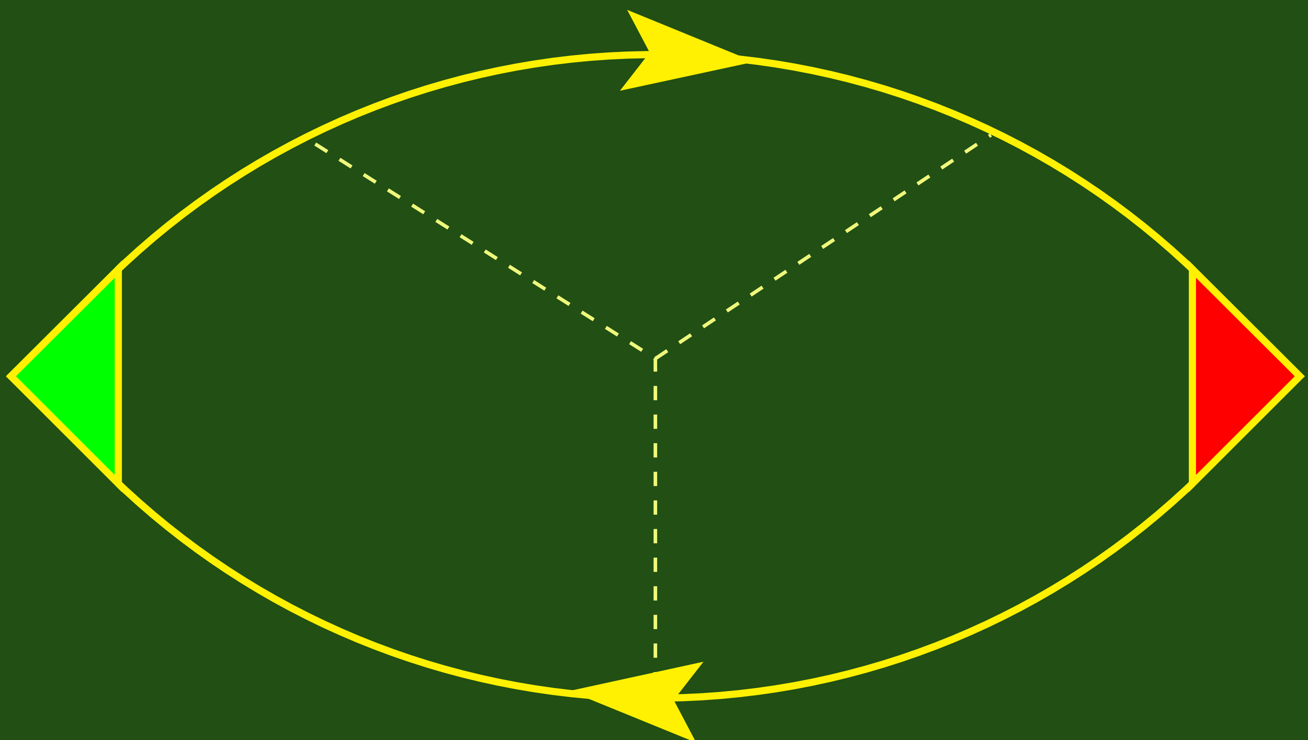
Due to the SO interaction, the asymptotic average trajectories of an electron (which are straight lines) scattered by a centered potential, do not cross in the center of the scatterer. This does not change the scattering angle $\Longrightarrow$ does not affect scattering crossection. Still it contributes to anomalous Hall effect.



see PRB24559 and cond-mat/0511310, 0603144.

Note about skew scattering

According to `arXiv:cond-mat/0506189`, taking skew scattering into account corresponds to considering diagrams (in the disorder averaging technique) where a disorder line has more than two ends:



The same J. Sinova said in Pohang.

Such diagrams are also drawn on fig. [3]5.

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