Journal Club by Oleg Chalaev

Středa-like formula in spin Hall effect

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good-old Středa

See [Stř82]: one can split Kubo formula for conductivity in two terms:

$$\sigma_{xy} = \sigma^{I}_{xy} + \sigma^{II}_{yx} \tag{1}$$

- classical term – contribution from electrons on the Fermi level:

$$\sigma^{I}_{xy} = \frac{e^2}{2} i\hbar \,\mathrm{Tr}[v_x G_{\mathrm{R}}(E_{\mathrm{F}}) v_y \delta(E_{\mathrm{F}} - \hat{H}) - v_x \delta(E_{\mathrm{F}} - \hat{H}) v_y G_{\mathrm{A}}(E_{\mathrm{F}})]$$

- quantum term: approaches zero in the classical limit:

$$\sigma^{II}_{xy} = -\sigma^{II}_{yx} = ec \frac{\partial N(E)}{\partial B}, \quad N(E) = \int_{-E_{\mathrm{F}}}^{E} \mathrm{Tr}\, \delta(\eta - \hat{H}) \mathrm{d}\eta.$$

The separation (1) claimed to be used in many quantum Hall papers. A nicely-written example: [CB01].

What about quantum spin-Hall effect?

About quantum spin-Hall effect

- No magnetic field needed.
- No disorder needed.
- "Landau levels" in the density of states are achieved using spin-orbit plus $\vec{E}$ (or strain): SOI $[\vec{p} \times \vec{E}]\vec{\sigma}$ leads to the term in the Hamilonian

$$E\sigma_z(xp_y - y_p x) = -\frac{eB_{\text{eff}}}{2mc}(xp_y - y_p x), \quad B_{\text{eff}} = \frac{2mc}{e}\sigma_z.$$

- No oscillations in $\sigma_{xx} = 0$.

A clearly written article is `cond-mat/0504147`.
See also `cond-mat/0411737`, `0506589`, `0509709`.

What happens with Středa in case of spin-Hall

- instead of $N(E)$, now we have magnetization s_z

$$\sigma_{xy}^{II} = -\sigma_{yx}^{II} = \frac{\partial s_z}{\partial B},$$

  so that instead of the spin current we can calculate magnetization.
- additional “non-conserved” correction to σ_{xy}^{II} appears.

Application example

In case of cond-mat/0504147:
we have an insulator $\Longrightarrow \sigma^{I}_{xy} = 0$. Also non-conserved part of σ^{II}_{xy} is zero, since SOI conserves s_z.

$$\delta\sigma = \frac{e}{2\pi}$$

$$N_{\uparrow\downarrow} = e(B_{\mathrm{eff}} \pm B)/h$$

$$\Longrightarrow \sigma^{II}_{xy} = -\frac{\partial s_z}{\partial B} = \frac{e}{2\pi}.$$

this document is available here.

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