Journal Club by Oleg Chalaev

Nonequilibrium Luttinger Liquid: Zero-Bias Anomaly and Dephasing

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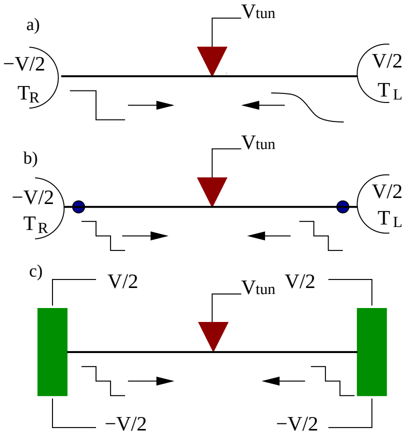
Possible experimental setup

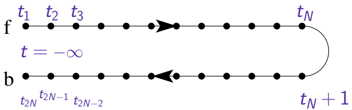


General idea and methods

- We all know that tunneling DoS has a dip due to electron-electron interaction.
- We strongly expect that there will be two dips in case of a 2-step distribution (known for diffusive systems).
- Let the expectations come true...

Methods of calculation: Keldysh technique in path-integral formulation [Kamenev'04].

Evolution over the closed contour



Evolution operator: $i\hbar \frac{\partial U_C(t,t_0)}{\partial t} = H_t U_C(t,t_0), \;\; U_C(t_0,t_0) \equiv 1,$

$$\mathrm{Tr}\, \hat{U}_C = \int D[\bar{\phi}(t)\phi(t)] \exp\left[i\bar{\phi}^T g^{-1}\phi\right] = \det\left(-ig^{-1}\right),$$

where, e.g., for $N = 4$

$$\frac{i}{\hbar} s(\bar{\phi},\phi) = i\bar{\phi}^T g^{-1}\phi, \quad \bar{\phi}^T = (\bar{\phi}_1 \dots \bar{\phi}_N, \bar{\phi}_{N+1} \dots \bar{\phi}_{2N}), \quad \phi^T = (\phi_1 \dots \phi_N, \phi_{N+1} \dots \phi_{2N}),$$

$$ig^{-1} = \left[\begin{array}{cccc|cccc}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 - \frac{i\delta_t}{\hbar}H & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 - \frac{i\delta_t}{\hbar}H & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 - \frac{i\delta_t}{\hbar}H & -1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 + \frac{i\delta_t}{\hbar}H & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 + \frac{i\delta_t}{\hbar}H & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 + \frac{i\delta_t}{\hbar}H & -1
\end{array}\right],$$

Keldysh action I

The expectation value of a physical quantity $\hat{O}$

$$\bar{O} = \mathrm{Tr}\left[\hat{\rho}_t \hat{O}\right] = \mathrm{Tr}\left[\hat{\rho}_{t_0} \hat{O}_t\right] = T_C \left\{ \mathrm{Tr}\left[\hat{U}_C \hat{\rho}_{t_0} \hat{O}\right] \right\},$$

$$\hat{O}_t \equiv U_C^{\dagger}(t, t_0) \hat{O} U_C(t, t_0).$$

The density matrix $\hat{\rho}_{t_0}$ may be non-equilibrium.
We split the integration over the contour $\oint \mathrm{d}t$ in two usual integrations:

$$\phi_f(t_i) \stackrel{\mathrm{df}}{=} \phi_i, \quad \phi_b(t_i) \stackrel{\mathrm{df}}{=} \phi_{2N+1-i} \Longrightarrow$$

$$\frac{i}{\hbar} S[\bar{\phi}, \phi] = i \bar{\phi} G^{-1} \phi = \int_{-\infty}^{\infty} \left[\bar{\phi}_b \dot{\phi}_b + i \bar{\phi}_b H \phi_b - \bar{\phi}_f \dot{\phi}_f - i \bar{\phi}_f H \phi_f \right] \mathrm{d}t.$$

– This is how we get Keldysh matrix structure.

Keldysh action II

For free fermions:

$$S[\bar{\psi}, \psi] = \hbar \bar{\psi} G^{-1} \psi, \quad G = \begin{pmatrix} \hat{G}_{\mathrm{R}} & \hat{G}_{\mathrm{K}} \\ 0 & \hat{G}_{\mathrm{A}} \end{pmatrix}.$$

With the interaction:

$$S_{int} = \frac{1}{2} \oint \mathrm{d}t \int \mathrm{d}r \mathrm{d}r' U(r - r') \bar{\psi}_r \bar{\psi}_{r'} \psi_r \psi_{r'},$$

Introducing *real* Stratonovich-Hubbard field:

$$e^{\frac{i}{2} \oint \mathrm{d}t \int \mathrm{d}r \mathrm{d}r' U(r-r') \bar{\psi}_r \bar{\psi}_{r'} \psi_r \psi_{r'}} = \int \mathcal{D}\phi e^{i \oint \mathrm{d}t \left[\frac{1}{2} \int \mathrm{d}r \mathrm{d}r' \phi_r U^{-1}_{rr'} \phi_{r'} + \int dr \phi_r \bar{\psi}_r \psi_r \right]},$$

so now we can integrate over the Grassmann variables $(\bar{\psi}, \psi)$ and forget them forever.

Keldysh action III

After integrating away the fermions:

$$S = \int \mathrm{d}t \int \mathrm{d}r \mathrm{d}r' \phi U^{-1} \sigma_1 \phi + \mathrm{Tr} \log \left[1 + G \left(\phi_1 \sigma_0 + \phi_2 \sigma_1\right)\right]$$

Now we expand the log up to the 2nd order in ϕ and call the result RPA:

$$S_{RPA}[\phi] = \int \mathrm{d}t \mathrm{d}t' \int \mathrm{d}r \mathrm{d}r' \left[\phi \left(U^{-1} \sigma_1 - \Pi\right) \phi\right],$$

where Π is the *bare* polarization matrix.

In the 1D case

$$H = iv\left(\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L\right) + \frac{V}{2}(\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L)^2,$$

$$S[\psi, \phi] = i \sum_{\eta = R,L} \psi_\eta^* (\partial_\eta - \phi)\psi_\eta - \frac{1}{2}\phi V^{-1} \phi,$$

$$S_{RPA}[\phi] = \int \mathrm{d}t\mathrm{d}t' \int \mathrm{d}r\mathrm{d}r' \phi \Pi \phi,$$

where the non-equilibrium polarization is $\Pi = \Pi_R + \Pi_L$, with

$$\Pi^r_{R,L} = -\frac{1}{2\pi}\frac{q}{\omega_+ \mp v_F q}\ , \ \Pi^a_{R,L} = -\frac{1}{2\pi}\frac{q}{\omega_- \mp v_F q},$$

$$\Pi^K_\eta = (\Pi^r_\eta - \Pi^a_\eta) B^v_\eta(\omega)\,, \quad \omega_\pm = \omega \pm i\delta,$$

$$B^v_\eta(\omega) = \frac{2}{\omega}\int_{-\infty}^{\infty} d\epsilon\, n_\eta(\epsilon)\left[2 - n_\eta(\epsilon - \omega) - n_\eta(\epsilon + \omega)\right].$$

Density of states I

Without interaction:

$$G(x;x') = \langle \bar{\psi}(x)\psi(x')\rangle = \int D[\bar{\psi}\psi] \exp\left[i\bar{\psi}G^{-1}\psi\right] \bar{\psi}(x)\psi(x').$$

In the presence of interaction:

$$G(x;x') = \langle \bar{\psi}(x)\psi(x')\rangle =$$
$$= \int D[\phi] \exp\left[i\phi V^{-1}\phi\right] \int D[\bar{\psi}\psi] \exp\left[i\bar{\psi}(G^{-1}+\phi)\psi\right] \bar{\psi}(x)\psi(x').$$

We denote $G_\phi = \int D[\bar{\psi}\psi] \exp\left[i\bar{\psi}(G^{-1}+\phi)\psi\right] \bar{\psi}(x)\psi(x')$.
In 1D G^{-1} contains $v_{\mathrm{F}}\partial_{\vec{r}}$ instead of $\Delta \Longrightarrow$ we get rid of ϕ in action by

$$\psi_\eta(x,t) \to \psi_\eta(x,t)e^{i\Theta_\eta(x,t)}, \quad \Theta_\eta = \sigma_0\theta + \sigma_1\bar{\theta}, \quad i\partial_\eta\vec{\theta}_\eta = \vec{\phi}$$
$$G_{\eta\phi}(x,t;x',t') = e^{i\Theta_\eta(t)}G_{\eta 0}(x,x';t-t')e^{-i\Theta_\eta(t')}.$$

Density of states II

$$\nu_\eta = \int \mathrm{d}t e^{-iE(t-t')} \left\langle \mathrm{Tr}\left[\sigma_3 G_{\eta\phi}\right]\right\rangle_\phi$$

Crucial: Θ depends linearly on ϕ, so the action remains quadratic in ϕ and the integration can still be performed:

$$\vec{\theta}_\eta = \mathcal{G}_{\eta 0}\vec{\phi}.$$

The rest of the calculation is straightforward:

$$\begin{aligned}\exp\left[i\Theta\right] &= \left[\sigma_0 \cos\bar{\theta} + i\sigma_1 \sin\bar{\theta}\right]\exp\left[i\theta\right] = \\ &= \frac{1}{2}\left\{\exp\left[i\left(\bar{\theta}+\theta\right)\right] + \exp\left[i\left(\bar{\theta}-\theta\right)\right]\right\}\sigma_0 + \\ &+ \frac{1}{2}\left\{\exp\left[i\left(\bar{\theta}+\theta\right)\right] + \exp\left[i\left(\bar{\theta}-\theta\right)\right]\right\}\sigma_1.\end{aligned}$$

Density of states III

The result of the Gaussian integration:

$$\frac{2\nu_\eta(\epsilon)}{\nu_0} = 1 + 2i\int_{-\infty}^{\infty} dt n_\eta(t) \exp\left(-I_{\theta\theta}^{(\eta)}\right)\sin(I_{\theta\bar{\theta}}^{(\eta)}),$$

$$I_{\theta\theta}^{(\eta)}(t) = \int (d\omega)(dq)(1-\cos(\omega t))\langle\theta\theta\rangle_{\omega,q}^{(\eta)} e^{-|\omega|/\Lambda},$$

$$I_{\theta\bar{\theta}}^{(\eta)} = 2\int (d\omega)(dq)\sin(\omega t)\langle\theta\bar{\theta}\rangle_{\omega,q}^{(\eta)} e^{-|\omega|/\Lambda},$$

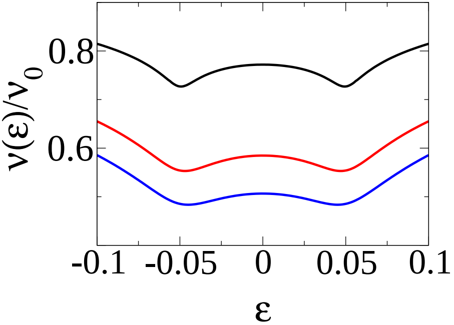
where Λ is an ultraviolet cutoff.

Non-perturbative correction to DoS

In equilibrium:

$$\nu_{\mathrm{eq}}(\epsilon) \sim \nu_0 \left(\epsilon/\Lambda\right)^{\gamma} .$$

For a two-step energy distribution:



Conclusion

- Two steps in the distribution function result in two DoS-dips.
- The way itself is the goal...

References

Alex Kamenev, cond-mat/0412296.