Journal Club by Oleg Chalaev

Three-particle collisions in quantum wires: Corrections to thermopower and conductance

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Three-particle interaction

Classical mechanics:

$$U = \sum_{i \le N} U_1(\vec{r}_i) + \sum_{i<j\le N} U_2(\vec{r}_i, \vec{r}_j) + \sum_{i<j<k\le N} U_3(\vec{r}_i, \vec{r}_j, \vec{r}_k).$$

Quantum mechanics:

$$\begin{aligned}\hat{U} \; &= \int U_1(\vec{r}\,)\hat{\psi}^\dagger(\vec{r}\,)\hat{\psi}(\vec{r}\,)\mathrm{d}^d r + \\ &+ \frac{1}{2}\iint U_2(\vec{r}_1,\vec{r}_2)\hat{\psi}^\dagger(\vec{r}_1)\hat{\psi}^\dagger(\vec{r}_2)\hat{\psi}(\vec{r}_2)\hat{\psi}(\vec{r}_1)\mathrm{d}^d r_1 \mathrm{d}^d r_2 + \\ &+ \frac{1}{6}\iiint U_3(\vec{r}_1,\vec{r}_2,\vec{r}_3)\hat{\psi}^\dagger(\vec{r}_1)\hat{\psi}^\dagger(\vec{r}_2)\hat{\psi}^\dagger(\vec{r}_3)\hat{\psi}(\vec{r}_3)\hat{\psi}(\vec{r}_2)\hat{\psi}(\vec{r}_1)\mathrm{d}^d r_1 \mathrm{d}^d r_2 \mathrm{d}^d r_3 + \\ &+ \ldots\end{aligned}$$

The distinction between U_2 and U_3 is clear in classics, but not in quantum mechanics.

Motivation

Questions about three particle interaction:

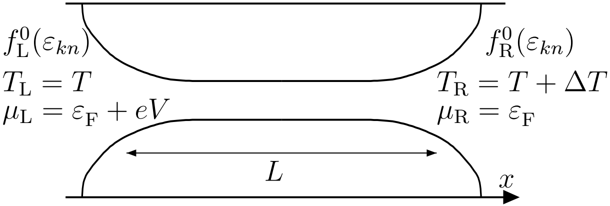
1. Does it exist?
2. Can it be important?
3. How can it be treated?
4. How strong is its effect on transport properties?

Answers:

1. Yes, if the particles are not classical “points”.
2. When the usual two particle interaction has no effect.
3. With standard Boltzmann equation approach.
4. $\exp[-E_{\mathrm{F}}/T]$, since it is governed by scattering in the bottom of the band.

Plan

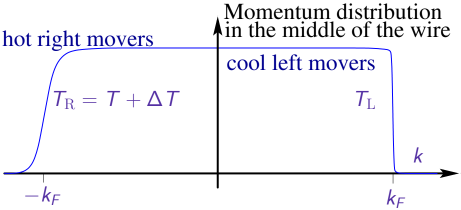
- The "usual" two-particle interaction does not affect conductivity
- Kinetic equation
- Scattering amplitude



Coductance and thermopower in the non-interacting limit

$$f_k^{(0)} = \begin{cases} f^0(\varepsilon_k - \mu_{\rm L}, T_{\rm L}) \equiv f_{\rm L}^0(\varepsilon_k) & \text{for } k > 0, \text{ (right movers)}, \\ f^0(\varepsilon_k - \mu_{\rm R}, T_{\rm R}) \equiv f_{\rm R}^0(\varepsilon_k) & \text{for } k < 0, \text{ (left movers)}. \end{cases}$$

$f_{\rm L}^0$ and $f_{\rm R}^0$ have different temperature and chemical potential.



$$\text{conductance}: \quad G^{(0)} = \frac{2e^2}{h}\left(1 - e^{-T_{\rm F}/T}\right),$$

$$\text{thermopower}: \quad S^{(0)} = \frac{k_{\rm B}}{e}\frac{T_{\rm F}}{T} e^{-T_{\rm F}/T}.$$

The effect of "usual" two-particle interaction

(if there is no disorder)

- In 2D and 3D "Umklapp" processes contribute to transport coefficients.
- In 1D momentum conservation $p_1 + p_2 = p'_1 + p'_2$ together with the energy conservation allows only interchange of particles for spectrum with positive curvature (i.e., $\forall k \;\; k\partial_k \varepsilon_k > 0$).
  (but not for linear spectrum, e.g., graphene)

$\Longrightarrow$ The 2-particle processes are ineffective in 1D.

Kinetic equation $\longrightarrow v_k \partial_x f_k(x) = \mathcal{I}_{kx}[f]$

$$\mathcal{I}_{k_1 x}[f] = \sum_{\sigma_2\sigma_3\sigma_{1'}\sigma_{2'}\sigma_{3'}} \sum_{\substack{k_2 k_3 \\ k_{1'} k_{2'} k_{3'}}} W_{123;1'2'3'} \times$$

$$\times \left[f_1 f_2 f_3 (1 - f_{1'})(1 - f_{2'})(1 - f_{3'}) - f_{1'} f_{2'} f_{3'} (1 - f_1)(1 - f_2)(1 - f_3) \right],$$

Boundary conditions for reflectionless contacts:

$$f_k(x=0) = f^0_{\mathrm{L}}(\varepsilon_k) \text{ for } k > 0, \quad f_k(x=L) = f^0_{\mathrm{R}}(\varepsilon_k) \text{ for } k < 0,$$

$$f^{(1)}_k(x) = \frac{x}{v_k} \mathcal{I}_k[f^{(0)}] \text{ for } k > 0, \quad f^{(1)}_k(x) = \frac{x - L}{v_k} \mathcal{I}_k[f^{(0)}] \text{ for } k < 0.$$

$\longleftarrow$ Questionable, since most relaxation occurs out of the wire!

Assumption: short wire

$$\Longrightarrow f = f^{(0)} + f^{(1)} + \ldots$$

Task: linearize collision integral and find $f^{(1)} \propto (V, \Delta T)$ to first order in W.

Calculating scattering amplitude W, or how to cheat Keldysh

The most important problem is solved not strictly, saying:
– Generalized Fermi rule, come on!

$$W_{123;1'2'3'} = \frac{2\pi}{\hbar}|\langle 1'2'3'|VG_0V|123\rangle_{\mathfrak{c}}|^2\delta(E_i - E_f),$$

$$G_0 = [E_i - H_0 + i0]^{-1},$$

$$V = \frac{1}{2L}\sum_{k_1 k_2 q}\sum_{\sigma_1\sigma_2} V_q c^\dagger_{k_1+q\sigma_1} c^\dagger_{k_2-q\sigma_2} c_{k_2\sigma_2} c_{k_1\sigma_1},$$

$$|123\rangle = c^\dagger_{k_1\sigma_1} c^\dagger_{k_2\sigma_2} c^\dagger_{k_3\sigma_3}|0\rangle,$$

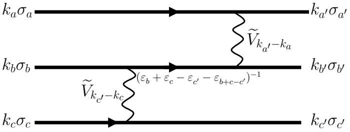
$$|1'2'3'\rangle = c^\dagger_{k_{1'}\sigma_{1'}} c^\dagger_{k_{2'}\sigma_{2'}} c^\dagger_{k_{3'}\sigma_{3'}}|0\rangle,$$

or, in other words, saying that we want the 3-particle Green function under the modulus.
(BTW, it is not completely retarded)

The diagrams for the 3-particle Green function

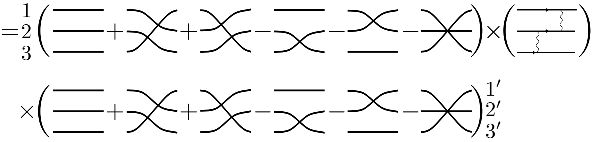
(a)

$$\frac{1}{4L^2}\frac{\widetilde{V}_{a'-a}\widetilde{V}_{c'-c}\ \delta_{a+b+c,a'+b'+c'}}{\varepsilon_b+\varepsilon_c-\varepsilon_{c'}-\varepsilon_{b+c-c'}+i\eta}\delta_{\sigma_{a'},\sigma_a}\delta_{\sigma_{b'},\sigma_b}\delta_{\sigma_{c'},\sigma_c} \equiv$$



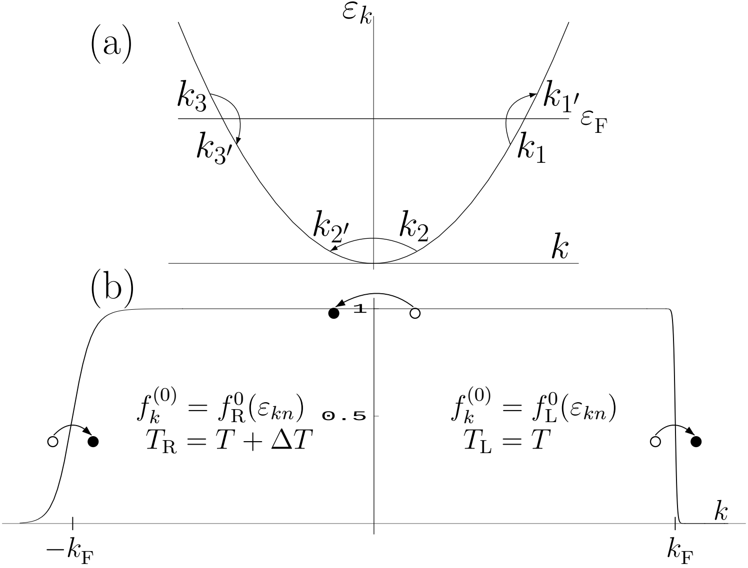
(b)

$$\langle 1'2'3'|VG_0V|123\rangle_{\mathfrak{c}} =$$



The result – MANY terms (however, with some symmetries)

Most important scatterings



– produces $f_1^0 f_2^0 f_3^0 (1 - f_{1'}^0)(1 - f_{2'}^0)(1 - f_{3'}^0) \propto e^{-E_F/T}$.

Final steps and results

$$\tilde{V}_q = V_0 \left[1 - \left(\frac{q}{q_0} \right)^2 + \mathcal{O}(q^4) \right],$$

where the parameter $q_0 \ll k_{\mathrm{F}}$ describes the screening due to the metallic gates.

Results: thermopower and conductance in the low temperature limit:

$$\begin{aligned} S = &\quad \frac{k_{\mathrm{B}}}{e} \frac{E_{\mathrm{F}}}{T} e^{-E_{\mathrm{F}}/T} \left[1 + \frac{L}{\ell_{\mathrm{eee}}} \right], \\ G = &\quad \frac{2e^2}{h} - \frac{2e^2}{h} e^{-E_{\mathrm{F}}/T} \left[1 + \frac{L}{\ell_{\mathrm{eee}}} \right], \\ \ell_{\mathrm{eee}}^{-1} &= \frac{8505}{2048\pi^3} \frac{(V_0 k_{\mathrm{F}})^4}{\varepsilon_{\mathrm{F}}^4} \left(\frac{k_{\mathrm{F}}}{q_0} \right)^4 \left(\frac{T}{E_{\mathrm{F}}} \right)^7 k_{\mathrm{F}}, \end{aligned}$$

Conclusions

- Three-particle collisions are essential in 1D.
- Unclear: how does this match Keldysh kinetic equation?
- The problem is interesting, but the authors could solve it more carefully.

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