Journal Club by Oleg Chalaev

Transport of interacting electrons through a potential barrier: nonperturbative RG approach

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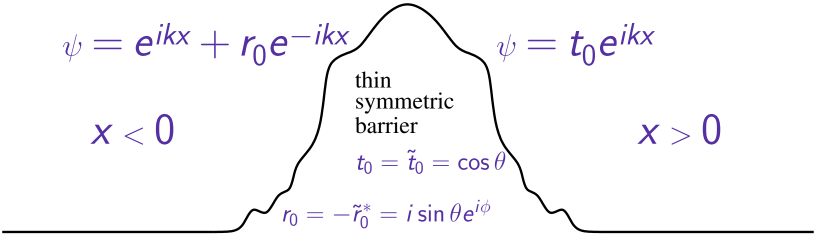
18.09.2007

cond-mat/0709.1562

1D wave functions in the presence of a barrier (non-equilibrium picture)



Without the interaction the conductance (spinless electrons)

$$G^{(0)} = \frac{e^2}{h} |t_0|^2.$$

Plugging the power of Green functions I

Asymptotic of the Green function for $x \to \infty$ [Yue et al.]:

$$G^{E_k}(x,y) = \frac{1}{iv_k} \begin{cases} te^{ik(x-y)}, y < 0, \\ e^{ik(x-y)} + re^{ik(x+y)}, y > 0. \end{cases}$$

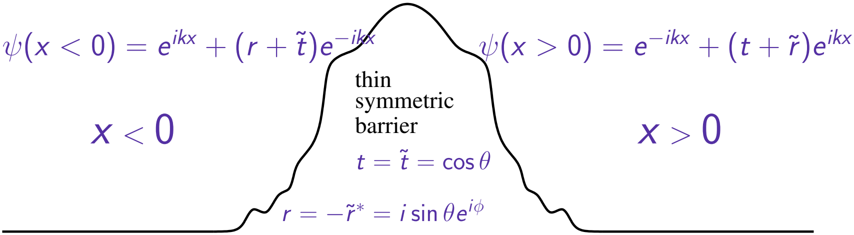
$\Longrightarrow$ we can use perturbation theory to calculate the GF, and extract $|t|^2$ from it asymptotic.

Two ways of calculating conductance:

- After we've calculated the Green function, we can extract the conductance from its asymptotic.
- Just apply the current operator to the Green's function – like in higher dimensions.

Plugging the power of Green functions II

Since we want to use the usual linear response approach (drawing bubbles), we introduce equilibrium states:



Then we use

$$\psi^+(x) = \left\{\Theta(-x)\left[\psi_1^+(x) + r\psi_1^+(-x) + \tilde{t}\psi_2^+(x)\right] + \Theta(x)\left[t\psi_1^+(x) + \tilde{r}\psi_2^+(-x) + \psi_2^+(x)\right]\right\}.$$

to construct Green functions – building blocks for the diagrams.

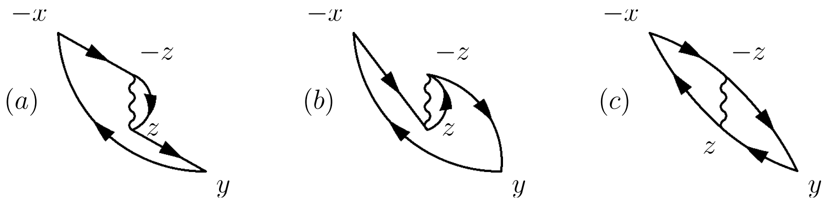
The interaction type

The Hamiltonian (product of reverse engineering):

$$H = \sum_k \frac{k^2}{2m} c_k^\dagger c_k + g_2 \int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y \psi_x^\dagger \psi_x \psi_{-y}^\dagger \psi_{-y}$$

- The interaction between electrons on the same side is not interesting (can be absorbed in v_F).
- Only the singlet channel is considered.

First order perturbation theory



We take into account only Fock terms, (singlet channel, momentum of the interaction line is small), we neglect triplet and Cooper channels because we don't like them...
First-order correction to the conductance:

$$G^{(1)} = -\frac{g_2}{4\pi}\sin^2(2\theta)\log\frac{L}{a},$$

where L, a are system size and the barrier width.

This is a non-perturbative problem

The potential barrier is renormalized by the Coulomb interaction [Yue et al.]:

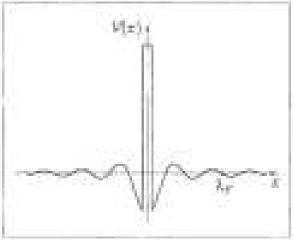


FIG. 1. Total scattering potential. The central peak is the bare potential of the barrier. The wings represent the Friedel oscillation induced by the barrier.

All interaction corrections logarithmically diverge
$\Longrightarrow$ one should try RG!

The RG transformation

- close to the barrier we don't feel its renormalization.
- On every RG step we move further away from the barrier

The transmission amplitude is changed on each step:

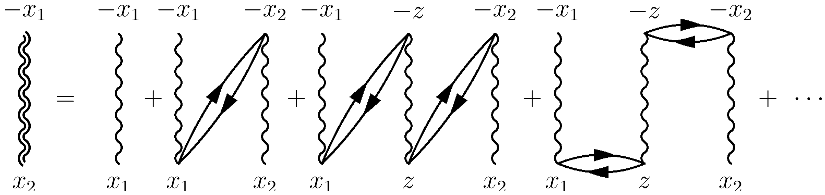
$$n, \delta n \in \mathbb{N}, n \gg 1, \quad \forall \delta n \ll n \quad t_{n+\delta n} = t_n + f(t_n) \ln \Lambda_0^{\delta n},$$

$$n \leftrightarrow z \equiv \log_{\Lambda_0} \Lambda, \quad \Lambda = \Lambda_0^{\delta n}, \quad \delta n \leftrightarrow \delta z,$$

$$z, \delta z \in \mathbb{R} \quad t_{z+\delta z} = t_z + f(t_z)\delta z \Rightarrow \frac{\mathrm{d}t}{\mathrm{d}\log_{\Lambda_0} \Lambda} = f(t).$$

– we should find the fixed point. (Reached on $L \sim (k - k_F)$, where the Friedel oscillations are no more important)

Summing the RPA series



$$G^{(L)} = -(1 - Y^2)\frac{g}{1 + \sqrt{1 - g^2} + gY} \log \frac{T_0}{T}.$$

where $g = g_2/(2\pi)$ and $Y = 2G - 1$.
$T_0 = 1/a$, $T = \min(L, v_F/T)$.

RG equation

$$G^{(L)} = -(1 - Y^2)\frac{g}{1 + \sqrt{1 - g^2} + gY}\log\frac{T_0}{T}.$$

$$\frac{dY}{d\Lambda} = -\frac{2g(1 - Y^2)}{1 + \sqrt{1 - g^2} + gY} = \beta(Y)$$

$$\frac{G^K}{(1 - G)} = \left(\frac{T}{T_0}\right)^{2(1-K)}\frac{|t|^{2K}}{|r|^2}, \quad K = \sqrt{\frac{1 - g}{1 + g}}.$$

Results

Repulsive interaction – conductance vanishes at $T \to 0$:

$$G = \left(\frac{T}{T_0}\right)^{2(\frac{1}{K}-1)} \quad \frac{|t|^2}{|r|^{2/K}} \ ,$$

Attractive interaction – barrier disappears at $T \to 0$:

$$G = 1 - \left(\frac{T}{T_0}\right)^{2(K-1)} \quad \frac{|r|^2}{|t|^{2K}}.$$

Conclusions

- Pro: Advance over [Yue et al.] – arbitrary interaction strength.
- Contra: Part of the interaction is neglected without justification.

this document is available here.

References

Dongxiao Yue, L. I. Glazman and K. A. Matveev
Phys. Rev. B, **49**, 1966 (1994).

Useless equations / Isocharge and isospin

$$\psi^+(x) = \left\{\Theta(-x)\left[\psi_1^+(x) + r\psi_1^+(-x) + \tilde{t}\psi_2^+(x)\right] \right. \\ \left. + \Theta(x)\left[t\psi_1^+(x) + \tilde{r}\psi_2^+(-x) + \psi_2^+(x)\right]\right\}$$

where $\psi_{1,2}^+(x) = \int_0^\infty \frac{dk}{2\pi} e^{\pm ikx} c_{1,2k}^+$.

$$J_\mu(x) = \frac{1}{2}\Sigma_{\alpha,\beta=1,2}\psi_\alpha^+(\alpha x)\sigma_{\alpha\beta}^\mu\psi_\beta(\beta x)$$

Useless equations / The Hamiltonian

$$
\begin{aligned}
H_0 &= 2\pi v_F \int_0^\infty dx \left[J_0^2(-x) + J_0^2(x) + J_3^2(-x) + \tilde{J}_3^2(x) \right] \\
H_1 &= 2g_2 \int_a^\infty dx \left[J_0(-x) J_0(x) - J_3(-x) \tilde{J}_3(x) \right]
\end{aligned}
$$

Useless equations / Current and conductance

Charge density:

$$\rho(x) = J_0(-x) + J_0(x) + \mathrm{sgn}(x)\left[-J_3(-|x|) + \tilde{J}_3(|x|)\right] = \rho_c(x) + \rho_s(x)$$

Current density:

$$j(x) = v_F\left[J_0(x) - J_0(-x) + J_3(-|x|) + \tilde{J}_3(|x|)\right] = j_c(x) + j_s(x)$$

Conductance (complete mystery):

$$G(x,t) = -2\pi i \Theta(t)\left\langle\left[j_s(x,t)\;,\;\int_0^\infty dy\;\rho_s(y,0)\right]\right\rangle$$