Theory of Phonon Hall Effect in Paramagnetic Dielectrics

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. . . will probably be published soon. . .

Thermo-magneto-electric effects

see §[Abr87]6.2

$$\text{el. field} = \vec{E} = \sigma\vec{j} + R\left[\vec{H}\times\vec{j}\right] + Q\vec{\nabla}T + N\left[\vec{H}\times\vec{\nabla}T\right]$$

$$\text{heat flux} = \vec{q} = \Pi\vec{j} + NT\left[\vec{H}\times\vec{j}\right] - \chi\vec{\nabla}T + L\left[\vec{H}\times\vec{\nabla}T\right]$$

Thermo-magneto-electric effects

see §[Abr87]6.2

$$\text{el. field} = \vec{E} = \sigma\vec{j} + R\left[\vec{H} \times \vec{j}\right]$$

$\nabla T = 0$, R= Hall resistivity

Thermo-magneto-electric effects

see §[Abr87]6.2

$$\text{el. field} = \vec{E} = \sigma\vec{j} + R\left[\vec{H} \times \vec{j}\right] + Q\vec{\nabla}T$$
$$0 = NT\left[\vec{H} \times \vec{j}\right] - \chi\vec{\nabla}T$$

$q = 0$, adiabatic Hall effect: $E_y = (R + QNT/\chi)Hj_x$

Thermo-magneto-electric effects

see §[Abr87]6.2

$$\text{heat flux} = \vec{q} = \Pi\vec{j} + NT\left[\vec{H}\times\vec{j}\right] - \chi\vec{\nabla}T + L\left[\vec{H}\times\vec{\nabla}T\right]$$

$q_y = 0, j_y = 0 \Longrightarrow \frac{\partial T}{\partial y} = \frac{NT}{\chi}Hj_x \longleftarrow$ Ettingshausen effect

Thermo-magneto-electric effects

see §[Abr87]6.2

$$\text{heat flux} = \vec{q} = \Pi\vec{j} + NT\left[\vec{H}\times\vec{j}\right] - \chi\vec{\nabla}T + L\left[\vec{H}\times\vec{\nabla}T\right]$$

$j_x = 0, j_y = 0, q_y = 0 \Longrightarrow \frac{\partial T}{\partial y} = \frac{L}{\chi}H\frac{\partial T}{\partial x}$, Leduc-Righi effect

Thermo-magneto-electric effects

see §[Abr87]6.2

$$\text{el. field} = \vec{E} = \sigma\vec{j} + R\left[\vec{H}\times\vec{j}\right] + Q\vec{\nabla}T + N\left[\vec{H}\times\vec{\nabla}T\right]$$

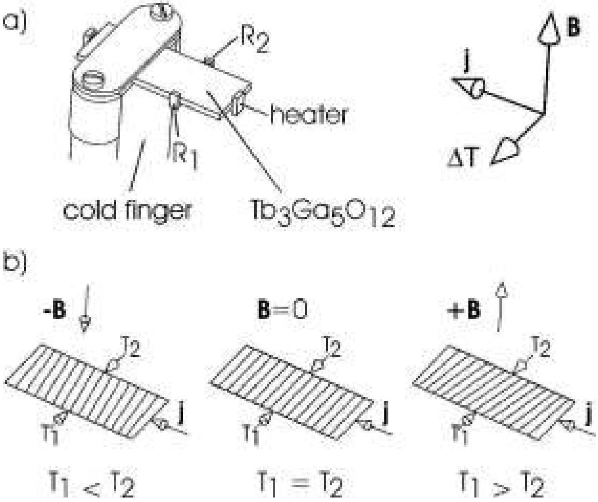
$$\text{heat flux} = \vec{q} = \Pi\vec{j} + NT\left[\vec{H}\times\vec{j}\right] - \chi\vec{\nabla}T + L\left[\vec{H}\times\vec{\nabla}T\right]$$

In insulators there is no Leduc-Righi effect (due to electrons). What about phonons?

Phonons do not interact with magnetic field directly $\Longrightarrow$we need unusual terms in the Hamiltonian

Experiment: Leduc-Righi effect for insulators

See [SRW05].
material: $Tb_3Ga_5O_{12}$ – dielectric, cubic, paramagnetic.



Experiment: Leduc-Righi effect for insulators

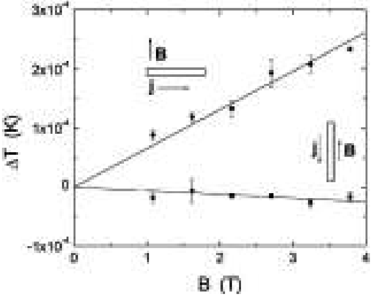
See [SRW05].
material: $Tb_3Ga_5O_{12}$ – dielectric, cubic, paramagnetic.

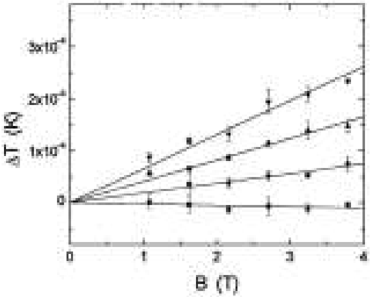
- immersed in ^{4}He-bath: $T = 5.45\text{K} \Longrightarrow$ there is NO optical phonons.
- producing temperature gradient via electrical heater
- thermometers: thermally activated hoping.
- eliminating thermometers' misaligment and even in $\vec{H}$ magnetoresistance effect:

$$\Delta T_{\perp} = \frac{1}{2s}\left[\Delta R(+B, \Delta T_{+}) - \Delta R(-B, \Delta T_{-})\right]$$

- phonon transport is diffusive, i.e., mean free path $\ll$ sample size.
- thermomagnetoresistance $\Longrightarrow$ magn. field affects phonon scattering

Experimental results





Spin-Phonon interactions: (pseudo)spin of an ion interacts with phonon

Phonons do not interact with magnetic field directly

Phonon excitations $\longleftrightarrow$ total angular momentum of an atom $\longleftrightarrow$ external magnetic field.

Here is the requested unusual term in the Hamiltonian – Raman interaction (mean field):

$$V = K \sum_m \mathbf{M} \cdot \mathbf{\Omega}_m$$

m=atom number. After quantisation:

$$V = \frac{1}{2} \sum_{\mathbf{q},\sigma,\sigma'} \Delta_{q\sigma\sigma'} \sqrt{\frac{\omega_{q\sigma'}}{\omega_{q\sigma}}} (a_{-q\sigma} + a^\dagger_{q\sigma})(a_{q\sigma'} - a^\dagger_{-q\sigma'})$$

where $\Delta_{q\sigma\sigma'} = -i\hbar K \mathbf{M} \cdot (\hat{\mathbf{e}}^*_{q\sigma} \times \hat{\mathbf{e}}_{q\sigma'})$

Thermal current operator

$$\mathbf{J}_{\mathrm{E}} = \frac{1}{2\mathcal{V}} \sum_{mn\alpha\beta} (\mathbf{R}_m - \mathbf{R}_n)\Phi^{\alpha\beta}(\mathbf{R}_m - \mathbf{R}_n) u_m^\alpha v_n^\beta \ ,$$

where u_m^α and v_m^α with $\alpha = x$, y and z are the α-components of the center-of-mass displacement $\mathbf{u}_m$ and velocity $\mathbf{v}_m$ of the m-th unit cell, respectively, and $\Phi^{\alpha\beta}(\mathbf{R}_m - \mathbf{R}_n)$ are the stiffness matrix elements of the lattice with $\mathbf{R}_m$ the equilibrium position of the unit cell.

Serious question: with Raman interaction included, the continuity equation for the heat flux [Har63] may not hold any more and then we have problems in how to define heat flux — just like in case of spin current (which is not conserved in systems with SOI). The authors did not mention this problem, and did not explain their derivation of the heat flux. May be they just wanted to hide this problem from the referee. . . Did they cheat?

Thermal current operator – continued

Modified due to spin-phonon interactions. $\mathbf{J}_E = \mathbf{J}_E^{(0)} + \mathbf{J}_E^{(1)}$, where

$$\mathbf{J}_E^{(0)} = \frac{1}{2\mathcal{V}} \sum_{\mathbf{q},\sigma,\sigma'} \mathbf{j}_{q\sigma\sigma'} \sqrt{\frac{\omega_{q\sigma'}}{\omega_{q\sigma}}} (a_{-q\sigma} + a^\dagger_{q\sigma}) \times (a_{q\sigma'} - a^\dagger_{-q\sigma'}) \ ,$$

$$\mathbf{J}_E^{(1)} = \frac{1}{2\mathcal{V}} \sum_{\mathbf{q},\sigma,\sigma',\sigma''} \mathbf{j}_{q\sigma\sigma''} \left(\frac{\Delta_{q\sigma''\sigma'}}{\hbar\sqrt{\omega_{q\sigma}\omega_{q\sigma'}}} \right) \times (a_{-q\sigma} + a^\dagger_{q\sigma})(a_{q\sigma'} + a^\dagger_{-q\sigma'}),$$

with

$$\mathbf{j}_{q\sigma\sigma'} = \hbar\omega_{q\sigma}\delta_{\sigma\sigma'}\nabla_q\omega_{q\sigma} + \frac{\hbar}{4}(\omega^2_{q\sigma} - \omega^2_{q\sigma'}) \times \\ \times \sum_\alpha \left[(\nabla_q \hat{\mathbf{e}}^{*\alpha}_{q\sigma})\hat{\mathbf{e}}^{\alpha}_{q\sigma'} - \hat{\mathbf{e}}^{*\alpha}_{q\sigma}(\nabla_q \hat{\mathbf{e}}^{\alpha}_{q\sigma'}) \right] \ .$$

Here, $\mathbf{J}_E^{(1)}$ comes from the Raman interaction.

Linear responce thermal current

Note: the derivation is not similar to the charge conductivity!

Does any one know simple and clear derivations?

A long (but clear) derivation is available in [Ф. М. Куни81]. (see also Kubo's handwaving in ([KTH85]4.6.48))

$$\kappa_{xy} = \frac{\mathcal{V}}{T}\int_0^{\hbar/k_{\mathrm{B}}T} d\lambda \int_0^{\infty} dt \langle J_{\mathrm{E}}^x(-i\lambda) J_{\mathrm{E}}^y(t)\rangle$$

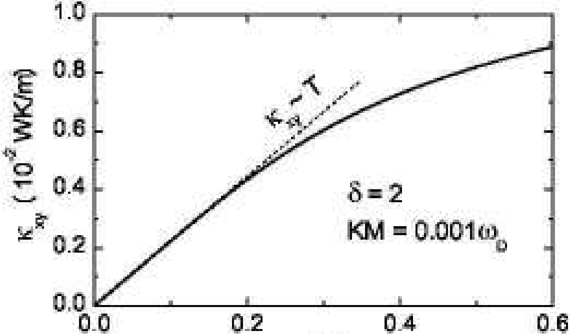
where J_{E}^x is the $x-$component of the energy flux operator $\mathbf{J}_{\mathrm{E}}$ of the phonons, and $\mathbf{J}_{\mathrm{E}}(t) = e^{iHt/\hbar}\mathbf{J}_{\mathrm{E}}e^{-iHt/\hbar}$.

Result

$$\kappa_{xy} = \frac{\gamma k_B KM}{2\pi^2 \bar{c}_s} \left(\frac{k_B T}{\hbar}\right) \int_0^{\Theta_D/T} \frac{x}{e^x - 1} dx \;, \qquad (1)$$

where $\gamma = (5-\delta)(1+\delta)^4/[4\delta^2(9+18\delta^3)^{1/3}]$ with $\delta = c_L/c_T$, $\bar{c}_s$ is the average sound speed defined by $3/\bar{c}_s^3 = (1/c_L^3 + 2/c_T^3)$ and $\Theta_D = \hbar\omega_D/k_B = (6\pi^2/\nu_0)^{1/3}\hbar\bar{c}_s/k_B$ is the Debye temperature with ν_0 the volume of a unit cell.

comparison with experiment:



Conclusions

theory fits experiment well $\Longrightarrow$ everyone is happy

this document is available here.

Alexey Alexeevich Abrikosov.
Fundamentals of the Theory of Metals.
Nauka, 1987.
см. DVD№5(по-русски).

R. J. Hardy.
Energy-flux operator for a lattice.
Phys. Rev., 132:168–177, 1963.
см. DVD№5.

A. S. Ioselevich and H. Capellmann.
Strongly correlated spin-phonon systems: A scenario for heavy fermions.
Phys. Rev. B, 51:11446–11462, May 1995.
см. DVD№5.

R. Kubo, M. Toda, and N. Hashitsume.
Statistical Physics II: Nonequilibrium statistical mechanics.
Springer series in solid state physics. Springer, 1985.

см. DVD№5.

C. Strohm, G. L. J. A. Rikken, and P. Wyder.
Phenomenological evidence for the phonon hall effect.
Phys. Rev. Lett., 95:155901, October 2005.

J. M. Ziman.
Electrons and Phonons. The Theory of Transport Phenomena in Solids.
The int. series of monographs on physics. Изд. ин. лит-ры., 1962.
см. DVD№5 Дж. Займан «Электроны и фононы. Явления переноса в твёрдых телах».

Ф. М. Куни.
Статистическая физика и термодинамика.
Наука, Москва, 1981.
см. DVD№5.

Help desk

Non-standart thermoconductivity in dielectrics: §[Zim62]8.8.

Raman interaction=coupling between phonons and localized spins, see [IC95]