Journal Club by Oleg Chalaev

Cooper pair turbulence in atomic Fermi gases

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Reminder: Parametric resonance

$\ddot{x} + sp(t)x = 0, \quad \forall t \; p(t+T) = p(t)$

Stability analysis is known for the simplified case:

$p(t) = 1 + \epsilon f(t), \quad \epsilon \ll 1, \quad s > 0.$

– Determined by Fourier harmonics of $f(t)$.

Simplest example – Mathieu equation:

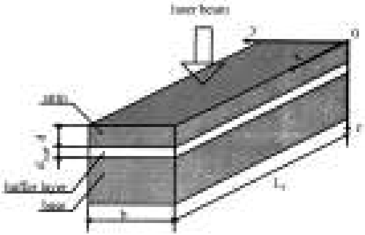
$$\ddot{x} + \omega_0^2(1 + \epsilon \cos t)x = 0$$

Instability zone: $\omega_0 \in \left[\frac{1}{2} - \frac{\epsilon}{8} + O(\epsilon^2), \frac{1}{2} + \frac{\epsilon}{8} + O(\epsilon^2)\right]$.

Earlier paper

See [Fomin, Shalaev& Shantsev '97]: Consider superconducting waveguide: The amplitude of the current J satisfies wave equation:

$$\frac{\partial^2 J(t)}{\partial t^2} + [\omega(\lambda(t))]^2 J(t) = 0.$$



where λ is the penetration depth of magnetic field. Modulating $\lambda = \lambda(T)$ with a laser, we induce parametric resonance (current waves).

The resonant frequency is determined by the geometry of the *macroscopic* sample.

Microscopic consideration –
– Dzero, Yuzbashyan & Altshuler

Bogoliubov – de Gennes microscopic equations:

$$i\dot{u}_{\vec{p}}(\vec{r},t) = \hat{\xi} u_{\vec{p}}(\vec{r},t) + \Delta(\vec{r},t) v_{\vec{p}}(\vec{r},t), \quad \hat{\xi} = -\vec{\nabla}^2/2m - \mu$$

$$i\dot{v}_{\vec{p}}(\vec{r},t) = -\hat{\xi} v_{\vec{p}}(\vec{r},t) + \bar{\Delta}(\vec{r},t) u_{\vec{p}}(\vec{r},t), \quad \Delta = g \sum_{\vec{p}} u_{\vec{p}}(\vec{r},t) \bar{v}_{\vec{p}}(\vec{r},t).$$

Assume that Δ is time-dependent and analyze the stability.

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Assume that Δ is time-dependent and analyze the stability.
Derivation – from initial mean-field Hamiltonian:

$$H = (a^\dagger, a)\begin{pmatrix} h & \Delta \\ \Delta^\dagger & -h^T \end{pmatrix}\begin{pmatrix} a \\ a^\dagger \end{pmatrix} =$$

$$= (b^\dagger, b)\begin{pmatrix} U^\dagger & V^T \\ V^\dagger & U^T \end{pmatrix}\begin{pmatrix} h & \Delta \\ \Delta^\dagger & -h^T \end{pmatrix}\begin{pmatrix} U & V \\ V^* & U^* \end{pmatrix}\begin{pmatrix} b \\ b^\dagger \end{pmatrix} =$$

$$= (b^\dagger, b)\begin{pmatrix} \epsilon/2 & 0 \\ 0 & -\epsilon/2 \end{pmatrix}\begin{pmatrix} b \\ b^\dagger \end{pmatrix} \Longrightarrow \begin{cases} \epsilon/2 \cdot U = hU + \Delta V^*, \\ \epsilon/2 \cdot V^* = -h^T V^* + \Delta^\dagger U. \end{cases}$$

Criticism

$$i\dot{u}_{\vec{p}}(\vec{r},t) = \hat{\xi} u_{\vec{p}}(\vec{r},t) + \Delta(\vec{r},t) v_{\vec{p}}(\vec{r},t), \quad \hat{\xi} = -\vec{\nabla}^2/2m - \mu$$

$$i\dot{v}_{\vec{p}}(\vec{r},t) = -\hat{\xi} v_{\vec{p}}(\vec{r},t) + \bar{\Delta}(\vec{r},t) u_{\vec{p}}(\vec{r},t), \quad \Delta = g\sum_{\vec{p}} u_{\vec{p}}(\vec{r},t)\bar{v}_{\vec{p}}(\vec{r},t).$$

Criticism: Thus the authors assume that *in any macroscopic sample* the instability occurs under the same conditions (i.e., the macroscopic geometry does not matter).
This contradicts to

- My old paper: macroscopic geometry matters!
- Common sense of turbulence: a critical Reinolds number depends on the geometry of the tube where gas or liquid flows. Also: how can turbulence exist without viscosity (i.e., interaction)?

Stability analysis

Assume that $\Delta(t) = \Delta_s[1 + q\cos(2\Delta_s t)], \quad q \ll 1.$

$$i\dot{u}_{\vec{p}}(\vec{r},t) = \hat{\xi}u_{\vec{p}}(\vec{r},t) + \Delta(\vec{r},t)v_{\vec{p}}(\vec{r},t), \quad \hat{\xi} = -\vec{\nabla}^2/2m - \mu$$
$$i\dot{v}_{\vec{p}}(\vec{r},t) = -\hat{\xi}v_{\vec{p}}(\vec{r},t) + \bar{\Delta}(\vec{r},t)u_{\vec{p}}(\vec{r},t), \quad \Delta = g\sum_{\vec{p}} u_{\vec{p}}(\vec{r},t)\bar{v}_{\vec{p}}(\vec{r},t).$$

Recipe (following Landau&Lifshitz v. 1) – introduce
small inhomogeneous terms growing in time:

$$\begin{bmatrix} u_{\vec{p}}(\vec{r},t) \\ v_{\vec{p}}(\vec{r},t) \end{bmatrix} = \begin{bmatrix} U_{\vec{p}}(t) + \phi_{\vec{p}}(\vec{r},t) \\ V_{\vec{p}}(t) + \psi_{\vec{p}}(\vec{r},t) \end{bmatrix} e^{i\vec{p}\cdot\vec{r}}, \quad \Delta(\vec{r},t) = \Delta(t) + \delta\Delta(\vec{r},t).$$

$$\delta\Delta(\vec{k},t) = C_{\vec{k}}(t)e^{i\Delta_s t} + \widetilde{C}_{\vec{k}}(t)e^{-i\Delta_s t}, \quad \phi_{\vec{p}}(\vec{k},t) = a_{\vec{p}}e^{i\xi_{\vec{p}}t} + b_{\vec{p}}e^{-i\xi_{\vec{p}}t}$$
$$C_{\vec{k}}(t) = c_{\vec{k}}e^{\nu(t-t_0)}, \widetilde{C}_{\vec{k}}(t) = \tilde{c}_{\vec{k}}e^{\nu(t-t_0)},$$
$$a_{\vec{p}}(\vec{k},t) \rightarrow (a_{1,\vec{p}}(\vec{k})e^{i\Delta_s t} + a_{-1,\vec{p}}(\vec{k})e^{-i\Delta_s t})e^{\nu t},$$
$$b_{\vec{p}}(\vec{k},t) \rightarrow (b_{1,\vec{p}}(\vec{k})e^{i\Delta_s t} + b_{-1,\vec{p}}(\vec{k})e^{-i\Delta_s t})e^{\nu t}$$

Stability analysis

Following Landau&Lifshitz "Mechanics" (where BTW, the derivation is inconsistent, though the result is correct)

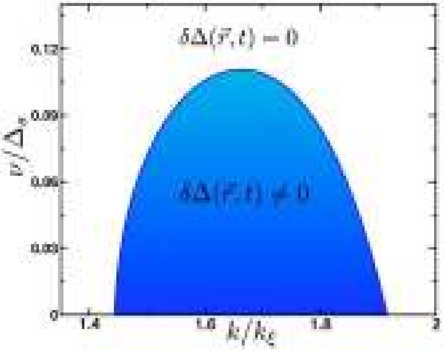

Figure: Region of the parametric instability of the homogeneous $\Delta(t)$ with respect to generation of the pairing modes with opposite momenta $(\vec{k}, -\vec{k})$. Instability growth rate is plotted for $q = 0.05$ ($\propto$ pumping intensity)

Post threshold state

Claim: In the end there will be inhomogeneous steady state.
The interaction terms (which we don't have in the Hamiltonian) will stop growth of parameters. These invisible terms will eat all the pumping power $\Longrightarrow$ we set $q \to 0$ in the equations and estimate the maximal $\nu_m \approx 2q\Delta_s$.

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Critisism:

1. How can we *ever* say something about the "post-threshhold state" using the linear analysis?
2. Something bad may happen to the mean-field theory if we assume that $\Delta(t) \propto \exp(i\Delta t)$. STOP DL: this is allowed.

Post threshold state

One possibility:

$$\Delta(\vec{r},t) = \Delta_s + \sqrt{q}\Delta_s c_s \sum_{|\vec{k}|=k_s} e^{i\vec{k}\cdot\vec{r}} \times \left[e^{i(\alpha_{\vec{k}}+\Delta_s t)} + w_s e^{i(\tilde{\alpha}_{\vec{k}}-\Delta_s t)}\right],$$

– leads to the oscillating supercurrent:

$$\vec{j}_s(\vec{r},t) \propto \hat{e}_r \sqrt{q}\cos(\Delta_s t)[k_s r\cos(k_s r) - \sin(k_s r)]/(k_s r)^2$$

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Another possibility:

$$\Delta(\vec{r},t) = \Delta_s + \frac{\sqrt{q}\Delta_s c_s \sin(k_s|\vec{r}-\vec{r}_0|)}{k_s|\vec{r}-\vec{r}_0|}A(t),$$

$$A(t) = e^{i(\frac{\Psi_s}{2}+\Delta_s t)} + w_s e^{i(\frac{\widetilde{\Psi}_s}{2}-\Delta_s t)}.$$

Conclusions

- A superconductor can be parametrically excited (well, who doubts?).
- To achieve the resonance, the order parameter must be pumped at high frequency.
- The authors claim to describe the “post-threshhold” state. (Seems impossible.)

this document is available here.

References

N.V.Fomin, O.L.Shalaev, and D.V.Shantsev
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