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cond-mat/0405523:

Non exponential quasiparticle decay and phase relaxation in low dimensional conductors

Gilles Montambaux and Eric Akkermans

problem:

How dephasing and phase decoherence due to electron-electron interaction in disordered system occur?

There are many times...

- Life time of the excitations in the Landau Fermi liquid theory, or inelastic time: Let $P(t)$ be the probability for a quasiparticle to stay in its initial state. Then $\tau_{\rm in}$ is the characteristic scale of $P(t)$.

- relaxation of the phase coherence: if $\Phi(t)$ = phase difference between time reversed trajectories [1], then τ_ϕ is the time scale of $\langle \exp\left[i\Phi(t)\right]\rangle$.

- time of the relaxation of the distribution function to the equilibrium:

$$\frac{\partial f_E}{\partial t} = -\frac{f_E - f_E^{\rm eq}}{\tau_f}$$

All times are of the same order(?)

Why τ_ϕ and $\tau_{\rm in}$ are important in mesoscopics?

dephasing kills the interference effects like:

- weak localization correction to the conductivity
- negative magnetoresistance in small $L_\phi \gtrsim L_H = \sqrt{\frac{\hbar c}{2eH}}$ magnetic fields, see B. L. Altshuler & A. G. Aronov, JETP letters, **33**, 499.

- so that τ_ϕ gives temperature dependence of these effects.

$\tau_{\rm in}$ serves as a cut-off, e.g. during the calculation of weak localization correction to conductivity in 2D [4].

Known results

- Landau IX: In the Fermi liquid $1/\tau_{\rm in} \propto (\epsilon - E_{\rm F})^2$.
- B. L. Altshuler & A. G. Aronov,
  JETP letters, **30**, 482 (1979):
  Landau theory ignores the finitness of the electron free path.
  In the disordered system $1/\tau_{\rm in} \propto (\epsilon - E_{\rm F})^{3/2}$, so that the quasiparticles' damping is stronger.
- Common point:
  the decay is exponential $\propto \exp[-t/\tau_{\rm in}]$

The key point

Why the decay is supposed to be exponential?
- because, like in a nuclear decay, every act of inelastic collision is independent from the others.

Montambaux & Akkermans:
this is not true because of the time-energy uncertainity $\Delta\omega\Delta t \gtrsim \hbar$ in the Fermi golden rule; in other words, the energy conservation during inelastic collisions is not exact for finite t.

The derivation of τ_ϕ

Writing the kinetic equation for f_E in Keldysh technique and linearising it, one obtains (see pp.348-349 [6])

$$\frac{1}{\tau_\phi} = \frac{T}{\nu}\int_0^T \mathrm{d}\omega \int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{1}{(Dq^2)^2+\omega^2}$$

then we say that in the integral $\omega \sim Dq^2$ so that $\int \mathrm{d}^d q \sim (\omega/D)^{d/2}$. The upper limit in $\int \mathrm{d}\omega$ is not important because it never diverges. Let us look on the lower limit. In 3D neither the lower limit diverges and no cut-off is required. Let us consider quasi-1D case, $d = 1$,

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} = \frac{1}{V}\sum_{\vec{q}} = \frac{1}{S}\int \frac{\mathrm{d}q}{2\pi}, \qquad \frac{1}{\tau_\phi} = \frac{T}{S\nu\sqrt{D}}\int_0^T \frac{\mathrm{d}\omega}{\omega^{3/2}}$$

usually they say that the lower cut-off is $1/\tau_\phi$, obtaining constant decay rate with $\tau_\phi \propto T^{-2/3}$. In [1] they say: the lower cut-off is $1/t$, not $1/\tau_\phi$!

Conclusions

- phase and excitations' relaxations in 2D are not exponential.
- the reason for this is taking into account the time-energy uncertainty in the Fermi golden rule.
- both τ_ϕ and τ_{in} have the same temperature dependence; also the corresponding relaxation laws are the same.

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this presentation has been prepared using only free software; see my presentations page for details.

References

Note: only references closely used in preparing this presentation are listed here. For deeper insight see Altshuler's papers between 1978-1984.

[1] Gilles Montambaux and Eric Akkermans, `arXiv:cond-mat/0405523` and `arXiv:cond-mat/0404361`.

[2] Pines, Nozieres, "The theory of quantum liquids", 1966, p. 62-63.

[3] B. L. Altshuler and A. G. Aronov "Electron-electron interaction in disordered systems" - in series edited by A. L. Efros and M. Pollak, Elsevier, 1985.

[4] Elihu Abrahams, P. W. Anderson, P. A. Lee and T. V. Ramakrishman, Phys. Rev. B, **24**, 6783 (1981).

[5] my informal notes.

[6] J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).