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Theoretical Solid-State Physics, Herbstsemester 2012

Blatt 8

Abgabe: 22. November, 12:00H (Treppenhaus 4. Stock)
Tutor: Samuel Aldana Zi.: 4.13

(1) **Zero-sound collective mode in charge-neutral Fermi gases** (6 Points)

In the lecture we showed that the conditions for the existence of the zero-sound collective mode in a Fermi gas with a hard-core interaction (i.e., an interaction potential whose Fourier transform is a q-independent constant: $V_q = V$) read

$$\mathrm{Re}\,\chi_0^{\mathrm{R}}(\mathbf{q},\omega) = \frac{1}{V} \quad , \quad \mathrm{Im}\,\chi_0^{\mathrm{R}}(\mathbf{q},\omega) = 0\,, \tag{1}$$

where the real part of the polarization function for a non-interacting Fermi gas is ($\hbar = 1$)

$$\mathrm{Re}\,\chi_0^{\mathrm{R}}(\mathbf{q},\omega) = -\frac{N_0}{2}\Big\{1 + \frac{m^2}{k_{\mathrm{F}}q^3}\left[4\varepsilon_{\mathrm{F}}\varepsilon_{\mathbf{q}} - (\varepsilon_{\mathbf{q}}+\omega)^2\right]\ln\left|\frac{\varepsilon_{\mathbf{q}} + qv_{\mathrm{F}} + \omega}{\varepsilon_{\mathbf{q}} - qv_{\mathrm{F}} + \omega}\right| + \frac{m^2}{2k_{\mathrm{F}}q^3}\left[4\varepsilon_{\mathrm{F}}\varepsilon_{\mathbf{q}} - (\varepsilon_{\mathbf{q}}-\omega)^2\right]\ln\left|\frac{\varepsilon_{\mathbf{q}} + qv_{\mathrm{F}} - \omega}{\varepsilon_{\mathbf{q}} - qv_{\mathrm{F}} - \omega}\right|\Big\}\,. \tag{2}$$

In the last equation $\varepsilon_{\mathbf{q}} = q^2/(2m)$ and $N_0 \equiv 3n/2\varepsilon_{\mathrm{F}} = mk_{\mathrm{F}}/\pi^2$ is the density of states at the Fermi energy in a non-interacting Fermi gas.

(a) By expanding $\mathrm{Re}\,\chi_0^{\mathrm{R}}(\mathbf{q},\omega)$ in Eq. (2) for small q, demonstrate explicitly that in the long-wavelength regime the conditions in Eq. (1) reduce to the equation

$$\frac{\omega_q}{qv_{\mathrm{F}}}\ln\left|\frac{\omega_q + qv_{\mathrm{F}}}{\omega_q - qv_{\mathrm{F}}}\right| = 1 + \frac{1}{N_0 V}\,. \tag{3}$$

(b) As also shown in the lecture, Eq. (3) has (for arbitrary coupling strength N_0V) one solution with linear dispersion

$$\omega_q = cq \tag{4}$$

where the zero-sound velocity c is determined by the implicit equation

$$c = v_{\mathrm{F}}\left(1 + \frac{1}{N_0V}\right)\frac{1}{\ln\left|\dfrac{c+v_{\mathrm{F}}}{c-v_{\mathrm{F}}}\right|}\,. \tag{5}$$

Show that in the weak-coupling limit ($N_0V \ll 1$) the last equation reduces to

$$c \simeq v_{\mathrm{F}}\left[1 + 2\,\exp(-\frac{1}{N_0V})\right]\,. \tag{6}$$

Try to give a physical interpretation of the last result.

(2) **Plasmon dispersion in an interacting electron gas** (4 Points)

As shown in the lecture, in the presence of the long-ranged Coulomb interaction the zero sound is replaced by a gapped plasmon mode. Using once again the expansion of $\mathrm{Re}\,\chi_0^{\mathrm{R}}(\mathbf{q},\omega)$ [see Eq. (2) in Problem 1] for small q and the conditions in Eq. (1) of Problem 1 (where V should now be replaced by $V_q \propto 4\pi/q^2$) show that the plasmon dispersion reads

$$\omega_q = \omega_p \left\{ 1 + \frac{3}{10} \left(\frac{q v_{\mathrm{F}}}{\omega_p} \right)^2 + \mathcal{O}(q^4) \right\} , \tag{7}$$

with $\omega_p = \omega_{q=0}$ being the plasma frequency:

$$\omega_p \equiv \sqrt{\frac{n e^2}{\epsilon_0 m}} \, . \tag{8}$$