Departement Physik, Universität Basel
Dr. Vladimir M. Stojanović (Zi. 4.16, Tel.: 267-3755, vladimir.stojanovic@unibas.ch)

Theoretical Solid-State Physics, Herbstsemester 2012

Blatt 6

Abgabe: 01.11.2012, 12:00H (Treppenhaus 4. Stock)
Tutor: Dr. Rakesh Tiwari

(1) **Noninteracting Green's function for topological insulators** (6 Points)

Recall the derivation of the noninteracting Green's function for a system of fermions. Consider now two-dimensional surface electrons (in the $x - y$ plane) described by the Hamiltonian

$$H_{\mathrm{TI}} = \hbar v_F \sum_{\mathbf{k},\alpha,\beta} c^\dagger_{\alpha\mathbf{k}} \mathbf{k} \cdot \boldsymbol{\sigma}_{\alpha\beta} c_{\beta\mathbf{k}}, \tag{1}$$

where the indices α and β represent the electron spin ($\uparrow$ or $\downarrow$), $\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y)$ is a vector of Pauli spin matrices, $\mathbf{k} \equiv (k_x, k_y)$ is the electron wave vector, and v_F the Fermi velocity. In this problem, we are interested in calculating the Green's function in the ground state ($T = 0$).

(a) Using the equations of motion for operators in the Heisenberg picture, determine the time-dependence of $c^\dagger_{\sigma\mathbf{k}}(t)$ and $c_{\sigma\mathbf{k}}(t')$ ($\sigma \in \{\uparrow, \downarrow\}$) for the noninteracting system described by H_{TI}.

(b) Using part (a), show that the noninteracting retarded Green's function

$$G^R_{\alpha\beta}(\mathbf{k}, t - t') \equiv -i\Theta(t - t')\langle [c^\dagger_{\alpha\mathbf{k}}(t), c_{\beta\mathbf{k}}(t')]\rangle \,, \tag{2}$$

which in this case is a 2×2 matrix in spin space, at $T = 0$ is given by

$$G^R(\mathbf{k}, t - t') = -i\Theta(t - t') e^{-iv_F \mathbf{k}\cdot\boldsymbol{\sigma}(t-t')} \,. \tag{3}$$

(c) Show that in the frequency domain

$$\begin{aligned} G^{\mathrm{R}}(\mathbf{k}, \omega) &\equiv \int d(t - t') e^{i(\omega+i\eta)(t-t')} G^R(\mathbf{k}, t - t') \\ &= \frac{v_F \mathbf{k}\cdot\boldsymbol{\sigma} - \omega - i\eta}{v_F^2 k^2 - \omega^2} \,, \end{aligned} \tag{4}$$

where $\eta \to 0^+$.

(2) **Momentum dependence of electron-phonon coupling** (4 Points)

Given below are three different types of short-range coupling of a single electron with Einstein phonons (or, more generally, dispersionless bosons) of frequency ω, on a one-dimensional lattice (lattice constant $\equiv 1$) with N sites. The coupling constants g, ϕ, and ϕ_b are dimensionless. For each of these couplings, find the equivalent momentum-space representation, i.e., transform the corresponding electron-phonon coupling Hamiltonian in real space to the form

$$H_{\text{e-ph}} = \frac{1}{\sqrt{N}} \sum_{k,q} \gamma(k,q) a^\dagger_{k+q} a_k (b^\dagger_{-q} + b_q) \,. \tag{5}$$

Here the a's and b's are the electron- and phonon operators, respectively, while k and q are the corresponding quasimomenta.

(a) Holstein-type (purely local) coupling:

$$H_{\text{e-ph}} = g\omega \sum_{i=1}^{N} a^\dagger_i a_i (b^\dagger_i + b_i) \tag{6}$$

(b) Su-Schrieffer-Heeger (Peierls-type) coupling:

$$H_{\text{e-ph}} = \phi\omega \sum_{i=1}^{N} (a^\dagger_{i+1} a_i + \text{h.c.})(b^\dagger_{i+1} + b_{i+1} - b^\dagger_i - b_i) \tag{7}$$

(c) "breathing" coupling (relevant in cuprate high-T_c superconductors):

$$H_{\text{e-ph}} = \phi_{\text{b}}\omega \sum_{i=1}^{N} a^\dagger_i a_i (b^\dagger_{i-1/2} + b_{i-1/2} - b^\dagger_{i+1/2} - b_{i+1/2}) \,. \tag{8}$$

Here, $i \pm 1/2$ refers to the fact that the Einstein oscillators are placed in the middle between two sites.

Then comment on the differences between these three types of electron-phonon interaction as far as the momentum dependence of the vertex function $\gamma(k,q)$ is concerned.