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Theoretical Solid-State Physics, Herbstsemester 2012

Blatt 5

Abgabe: 25. Oktober, 12:00H (Treppenhaus 4. Stock)
Tutor: Dr. Gerson J. Ferreira Zi.: 4.7b

(1) **Spectral function for bosons** (4 Points)
In the lecture we defined the spectral function $A(\nu,\omega) = -2\mathrm{Im}G^R(\nu,\omega)$. Show that in the case of bosons the greater ($G^>$) and lesser ($G^<$) Green's functions can be expressed through the spectral function as

$$\begin{aligned} iG^>(\nu,\omega) &= A(\nu,\omega)[1+n_B(\omega)] \\ iG^<(\nu,\omega) &= A(\nu,\omega)n_B(\omega)\,, \end{aligned}$$

where $n_B(\omega) \equiv [\exp(\beta\omega)-1]^{-1}$ is the Bose function.

(2) **Single level coupled to the continuum** (6 Points)

(a) Show that for a noninteracting system described by the Hamiltonian

$$H = \sum_{\nu\nu'} t_{\nu'\nu} a^\dagger_{\nu'} a_\nu$$

the Fourier-transformed ($t \to \omega + i\eta$) equation of motion for the retarded Green's function reads

$$\sum_{\nu''} [\delta_{\nu\nu''}(\omega+i\eta) - t_{\nu\nu''}] G_0^R(\nu''\nu';\omega) = \delta_{\nu\nu'}\,.$$

Here, the operators $a^\dagger$, a stand for either fermionic or bosonic creation and annihiliation operators.

(b) We now want to study a non-interacting electron gas ('continuum') coupled to a localized level, e.g., a single-level quantum dot. "Coupled" means that the electron can hop in and out of the dot. The Hamiltonian is

$$H = H_0 + H_l + H_{\mathrm{hyb}}\,,$$

where

$$H_0 = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}}$$

describes the electron gas,

$$H_l = \epsilon_l c^\dagger_l c_l$$

the localized level, and

$$H_{\mathrm{hyb}} = \sum_{\mathbf{k}} t_{\mathbf{k}} c^\dagger_l c_{\mathbf{k}} + \mathrm{h.c.}$$

describes the tunneling in and out of the dot ('hybridization').

The set of all quantum states ν in Eq. (2a) now runs over the wave vectors $\mathbf{k}$ plus l, the quantum number of the localized level. By setting first $\nu = \nu' = l$ and then $\nu = \mathbf{k}$, $\nu' = l$ in Eq. (2a), derive two coupled equations for $G^R(l, l; \omega)$ and $G^R(\mathbf{k}, l; \omega)$.

(c) Solve them and determine $G^R(l, l; \omega)$ explicitly. Interpret your solution. Calculate and discuss the spectral function A for the localized state. How is it changed by the coupling?