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Theoretical Solid-State Physics, Herbstsemester 2012

Blatt 4

Abgabe: 18. Oktober, 12:00H (Treppenhaus 4. Stock)
Tutor: Dr. Rakesh Tiwari

(1) **Spin-orbit coupling in the 2DEG** (4 Points)
In the lecture we discussed the general spin-orbit coupling term that originates from a nonrelativistic approximation to the Dirac equation:

$$H_{\mathrm{SO}} = -\Lambda\, \boldsymbol{\sigma} \cdot \mathbf{p} \times \boldsymbol{\nabla} V(\mathbf{r})\ ,$$

where $\Lambda \equiv \hbar/(2m_0 c)^2$ (m_0 is the free-electron mass, c the speed of light, and $\hbar$ Planck's constant), $\mathbf{p}$ is the electron momentum operator, $V(\mathbf{r})$ is the potential, and $\boldsymbol{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ the vector of Pauli spin matrices (the electron spin operator is $\mathbf{s} = \hbar\boldsymbol{\sigma}/2$).

(a) Across a planar interface between two different semiconductors, an electrostatic potential $\phi(|\mathbf{r}_\perp|)$ forms, where $|\mathbf{r}_\perp|$ is the distance of the point $\mathbf{r}$ to the interface. This potential gives rise to the Rashba spin-orbit interaction acting on electrons in the two-dimensional electron gas (2DEG) at the interface. Show that the corresponding spin-orbit Hamiltonian has the form

$$H_R \propto p_y\sigma_x - p_x\sigma_y$$

for an interface that lies in the $x-y$ plane.

(b) Determine the electron energy and the corresponding spinor wave functions in the 2DEG under the influence of the Rashba-type spin-orbit interaction, i.e., solve the eigenvalue problem of the Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m_0} + \frac{\alpha}{\hbar}(p_y\sigma_x - p_x\sigma_y)\ .$$

(c) The spin-orbit interaction can be thought of as an interaction of the electron with an effective magnetic field: $H_{\mathrm{SO}} \propto \boldsymbol{\sigma}\cdot\mathbf{B}_{\mathrm{eff}}$. Illustrate (plot or sketch) how the direction of $\mathbf{B}_{\mathrm{eff}}$ depends on the electron momentum both for the Rashba-type spin-orbit interaction [recall part (a)] and for the Dresselhaus spin-orbit interaction

$$H_D \propto p_x\sigma_x - p_y\sigma_y\ .$$

[Note: This form results for a AlGaAs/GaAs interface, when x, y, and z point along the main crystallographic directions (100), (010), and (001)].

(2) **Tight-binding model with Rashba-type spin-orbit interaction** (6 Points)
Let us consider a tight-binding Hamiltonian for electrons on a two-dimensional square lattice (lattice constant $a \equiv 1$) with the Rashba spin-orbit interaction (hereafter $\hbar = 1$):

$$H = -t \sum_{\langle n,n' \rangle, \alpha=\uparrow\downarrow} (c^\dagger_{n,\alpha} c_{n',\alpha} + \text{H.c.}) + V_s \sum_{n,\alpha,\beta} \left(i c^\dagger_{n,\alpha} \sigma_x^{\alpha\beta} c_{n+\hat{y},\beta} - i c^\dagger_{n,\alpha} \sigma_y^{\alpha\beta} c_{n+\hat{x},\beta} + \text{H.c.} \right) .$$

Here t is electronic hopping integral and V_s the strength of the Rashba-type spin-orbit interaction; $c^\dagger_{n,\alpha}$ ($c_{n,\alpha}$) creates (annihilates) an electron at site n with spin projection α, while $\sigma_x^{\alpha\beta}$, $\sigma_y^{\alpha\beta}$ designate the (α, β) component of the usual Pauli matrices. While n runs over all the lattice sites, $\langle n, n' \rangle$ indicates that only nearest-neighbor hopping is included; $n + \hat{x}$ ($n + \hat{y}$) denotes the nearest neighbor of site n in the x (y) direction.

(a) Show that the two electronic bands (the upper and lower Rashba bands) resulting from the last Hamiltonian have dispersions given by

$$E^{\pm}_{\mathbf{k}} = -2t[\cos(k_x) + \cos(k_y)] \pm 2V_s \sqrt{\sin^2(k_x) + \sin^2(k_y)}$$

and that the eigenvectors are

$$\Psi^{\pm}_{\mathbf{k}} = \frac{1}{\sqrt{2}} \left[c^\dagger_{\mathbf{k}\uparrow} \pm \frac{\sin(k_y) - i \sin(k_x)}{\sqrt{\sin^2(k_x) + \sin^2(k_y)}} \, c^\dagger_{\mathbf{k}\downarrow} \right] |0\rangle \, .$$

(b) What is the ground-state energy E_0 of H (the dispersion minima of $E^-_{\mathbf{k}}$) and what is its degeneracy ? What are the locations of these minimum points in the Brillouin zone? By expanding $E^-_{\mathbf{k}}$ around the band minima, show that the ratio of the effective mass (in the presence of spin-orbit interaction) and the bare band mass $m_0 \equiv (4t)^{-1}$ is given by

$$\frac{m_{\mathrm{SO}}}{m_0} = \frac{1}{\sqrt{1 + \dfrac{V_s^2}{2t^2}}} \, .$$

(c) The non-interacting electron density-of-states (DOS) is defined for each band as

$$D^{\pm}(E) = \sum_{\mathbf{k}} \delta(E - E^{\pm}_{\mathbf{k}}) \, .$$

Demonstrate that the lower-band DOS in the vicinity of its minima is given by:

$$D^-(E = E_0) = \begin{cases} (4\pi t)^{-1} & , \text{ for } V_s = 0 \, ; \\ \sqrt{2}\,(\pi V_s)^{-1} & , \text{ for } V_s \neq 0 \, . \end{cases}$$