Thermoelectricity in a junction between interacting cold atomic Fermi gases

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A gas of interacting ultracold fermions can be tuned into a strongly interacting regime using a Feshbach resonance. Here we theoretically study quasiparticle transport in a system of two reservoirs of interacting ultracold fermions on the BCS side of the BCS-BEC crossover coupled weakly via a tunnel junction. Using the generalized BCS theory we calculate the time evolution of the system that is assumed to be initially prepared in a non-equilibrium state characterized by a particle number imbalance or a temperature imbalance. A number of characteristic features like sharp peaks in quasiparticle currents, or transitions between the normal and superconducting states are found. We discuss signatures of the Seebeck and the Peltier effect and the resulting temperature difference of the two reservoirs as a function of the interaction parameter \((k_F a)^{-1}\). The Peltier effect may lead to an additional cooling mechanism for ultracold fermionic atoms.

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I. INTRODUCTION

Thermal transport is an important tool to investigate many-body systems. There is a variety of transport coefficients describing the heat carried by thermal currents as well as the voltages (in the case of charged particles) or chemical potential differences (in the case of neutral particles) induced by a thermal gradient (Seebeck effect). The inverse effect, the build-up of a thermal gradient by a particle current is of great practical importance (Peltier effect). These thermoelectric effects depend in sensitive ways on the excitation spectrum of the system close to the Fermi surface\(^1\). If the spectrum is particle-hole symmetric (as it is to a good approximation in the bulk of a metallic superconductor), the Seebeck effect vanishes. Breaking this symmetry in superconducting tunnel junctions allows for refrigeration\(^3\) and/or giant thermoelectric effects\(^4\)–\(^6\).

In recent years, transport in ultracold atomic gases has been investigated both theoretically\(^7\)–\(^10\),\(^16\) and in a number of experiments\(^11\)–\(^15\). Optical potentials were used to realize a narrow channel connecting two macroscopic reservoirs of neutral fermionic atoms to form an atomic analogue of a quantum mesoscopic device. Ohmic conduction in such a setup was observed\(^14\) as well as conductance plateaus at integer multiples of the conductance quantum \(1/h\) for a ballistic channel\(^14\). Tuning the interaction between the atoms by a magnetic field via a Feshbach resonance allowed to drive the system into the superfluid regime. The resulting drop of the resistance was observed experimentally\(^12\). Moreover, a quantum point contact between two superfluid reservoirs was realized\(^15\). Signatures of thermoelectric effects were observed in the normal state of these systems\(^13\). Several theoretical studies also examined mesoscopic transport\(^16\), thermoelectric effects\(^17\), and Peltier cooling in ultracold fermionic quantum gases\(^18\),\(^19\).

In this paper, we investigate the coupling of thermal and particle currents in a junction of two superfluids. The goal is to explore the possibility to realize dynamical heating and refrigeration phenomena around the phase transition. To this end, we consider two reservoirs of interacting ultracold atoms connected by a weak link that we model as a tunnel junction. The generalized BCS theory\(^20\) provides self-consistency equations for the gap parameter and the chemical potential as a function of the dimensionless interaction parameter \((k_F a)^{-1}\). We use the tunneling approach to describe quasiparticle transport in a system with a fixed number of particles and specify the initial particle and/or temperature imbalance of the two reservoirs. The resulting time evolution of the system shows a number of characteristic features: we find transitions between superfluid and normal states as well as signatures of the Peltier and Seebeck effects. In addition, there are peaks in the transport current that can be related to a resonant condition in the expression for the tunneling current.

The paper is organized as follows: In Sec. II we introduce a model Hamiltonian for the system consisting of two tunnel-coupled reservoirs as well as the self-consistency equations for the superconducting gap and the chemical potential in the generalized BCS theory. We also give expressions for the particle and the heat current. In Sec. III we calculate the time evolution of the system with a fixed total number of particles initially prepared with an imbalance in particle number and/or temperature. Finally, we conclude in Sec. IV.

II. MODEL

Our system, depicted in Fig. 1, consists of two reservoirs of interacting neutral fermionic atoms connected by a weak link that is modeled by a tunnel junction. Exper-
The tunneling Hamiltonian is represented by the two hyperfine states of the atom in the left (right) reservoir, \( \sigma \). The mean-field parameter \( \Delta(T) \) is introduced. The solution to these equations is shown in Fig. 2 as a function of temperature \( T \). Note that this mean-field critical temperature \( T_c \) is in fact the pairing temperature below which a significant number of fermions are bound in pairs. In the BCS limit \((k_F a)^{-1} \rightarrow -\infty\), the chemical potential \( \mu/\varepsilon_F \) → 1 and \( \Delta \) as well as \( T_c \) approach zero.

For the density of states (DOS) of a 3D Fermi gas in the normal state \( N(0) \propto \sqrt{\varepsilon} \) (neglecting the confining potential) which we used above, the integrals in Eqs. (5) and (6) converge and no cut-off energy needs to be introduced. The solution to these equations is shown in Fig. 2 as a function of temperature \( T \) and interaction parameter \((k_F a)^{-1}\). As the interaction parameter approaches the BCS limit, \((k_F a)^{-1} \ll -1\), the superconducting gap \( \Delta \) and critical temperature \( T_c \) are proportional to \( e^{-\pi/(2k_F a)} \) and \( \mu/\varepsilon_F \rightarrow 1 \) at \( T = 0^+ \). On the other hand, towards unitarity, where \((k_F a)^{-1} \rightarrow 0^-\), \( \Delta \) and \( T_c \) increase and \( \mu \) decreases.

FIG. 1: Two reservoirs of ultracold fermions connected via a tunnel junction allowing particle and heat transport. Each reservoir is characterized by the particle number \( N \) and temperature \( T \).

FIG. 2: (Color online) Solution for \( \Delta \) (lower blue surface) and \( \mu \) (upper orange surface) following from Eqs. (5) and (6) as a function of \((k_F a)^{-1}\) and \( T \). The mean-field critical temperature \( T_c \) is highlighted in the lower surface as a white curve. In the BCS limit \((k_F a)^{-1} \rightarrow -\infty\), the chemical potential \( \mu/\varepsilon_F \rightarrow 1 \) and \( \Delta \) as well as \( T_c \) approach zero.
current \(I\) and energy current \(I_\epsilon\) are defined as
\[
I = \frac{\partial (\hat{N}_L)}{\partial t} = i(\hat{N}_L, H)
\]
\[
I_\epsilon = \frac{\partial (\hat{H}_L)}{\partial t} = i(\hat{H}_L, H),
\]
where the angular brackets represent the thermodynamic average in the grandcanonical ensemble and \(\hat{N}_L = \sum_{\mu} c_{\mu L}^\dagger c_{\mu L}\) is the fermion number operator in the left reservoir. All the operators are in the Heisenberg picture. These expressions for the currents are general. The commutator on the right-hand side of Eq. (7) leads to terms describing quasiparticle tunneling and Cooper-pair tunneling, see e.g. Ref. 21 for the explicit evaluation of the particle current and the appendix of Ref. 22 for the evaluation of the heat current. In the following, we restrict ourselves to quasiparticle transport (i.e., ignore terms corresponding to Cooper-pair tunneling). Consequently, the expressions for the particle and heat current in the tunneling limit read
\[
I = I_{L \rightarrow R} - I_{R \rightarrow L} = \frac{2\pi|\eta|^2}{\hbar} V_L V_R \int_{-\infty}^\infty dE N_L(E) N_R(E) \left[ f_L(E) - f_R(E) \right]
\]
and
\[
I_Q = I_{Q,L \rightarrow R} - I_{Q,R \rightarrow L} = \frac{2\pi|\eta|^2}{\hbar} V_L V_R \int_{-\infty}^\infty dE N_L(E) N_R(E) \times \left[ (E - \mu_L) f_L(E)(1 - f_R(E)) - (E - \mu_R) f_R(E)(1 - f_L(E)) \right].
\]
Here, \(V_L(R)\) is the volume and \(f_L(R)(E)\) the Fermi function describing the left (right) reservoir. The superconducting density of states
\[
N_{L(R)}(E) = \text{Re} \{ N^0_{L(R)}(\epsilon) \} = \text{Re} \{ \frac{|E - \mu_{L(R)}|}{(E - \mu_{L(R)})^2 - \Delta^2_{L(R)}} \}
\]
contains the energy-dependent density of states \(N^0_{L(R)}\) of a normal 3-dimensional Fermi gas that can be expressed as
\[
N^0_{L(R)}(\epsilon) = \frac{1}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\epsilon} = \frac{1}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\mu_L + \text{sign}(E - \mu_L) \text{Re} \sqrt{(E - \mu_L)^2 - \Delta^2_{L(R)}}}.\]
The quasiparticle currents (8) and (9) can be understood already on a phenomenological level: the first term in Eq. (9) describes quasiparticles with energy \((E - \mu_L)\) tunneling with probability \(|\eta|^2\) from the left to right reservoir while the second term describes quasiparticles with energy \((E - \mu_R)\) tunneling from the right to left reservoir. Similar considerations lead to Eq. (8) for the particle current: the terms involving products of Fermi functions of the left and right reservoirs cancel.

### III. Time Evolution of the System

For finite reservoirs, which is the case we are studying here, a non-equilibrium initial state (like a temperature or particle number imbalance between the left and right reservoir) will induce time-dependent transport. To model this phenomenon we consider the balance equations for the particle number \(N_{L(R)}\) and energy \(E_{L(R)}\) in each reservoir that lead to
\[
\begin{align*}
\frac{\partial N_{L(R)}}{\partial t} &= \mp I \\
\frac{\partial E_{L(R)}}{\partial t} &= \frac{1}{C_{V_{L(R)}}} (I_Q + \mu_L I_{L \rightarrow R} - \mu_R I_{R \rightarrow L}).
\end{align*}
\]
Here, we used the relation between the energy of the left (right) reservoir and temperature change of the system at constant volume \(C_V = \partial E/\partial T\). The heat capacity in the (modified) BCS theory described by Eqs. (5) and (6) is given by
\[
C_V(T) = \frac{2}{T} \int_{-\infty}^{\infty} dE N(E) \left( -\frac{\partial f(E)}{\partial E} \right) \times \left( E^2 - T \frac{\partial \Delta^2}{\partial T} + T \text{sign}(E) \sqrt{E^2 - \Delta^2} \frac{\partial \mu}{\partial T} \right).
\]
In writing Eqs. (10) and (11), we have neglected number and energy fluctuations in the reservoirs which were shown to be small in the regime considered here.

To calculate the time evolution of the system, we proceed as follows: starting with \(N_{L(R)}(t) = N \pm \delta N/2\) and \(T_{L(R)}(t) = T \pm \delta T/2\) at time \(t\), we calculate the corresponding values of \(\mu_{L(R)}(t)\) and \(\Delta_{L(R)}(t)\) using Eqs. (5) and (6). Then, using the discretized form of Eq. (10), we obtain \(N_{L(R)}(t + \delta t)\) and \(T_{L(R)}(t + \delta t)\) at time \(t + \delta t\), and the procedure is iterated. The time evolution is hence uniquely determined by setting initial values of \(N^0_{L(R)}\), \(T^0_{L(R)}\) and \((k_B T/\mu)^{-1}\), where quantities with superscript 0 denote the values at time \(t = 0\). The interaction parameter on the right side follows from \((k_B T/\mu)^{-1}\) and \(N^0_{R}\). Note that in linear response in \(\delta N\) and \(\delta T\), assuming \(\Delta = \Delta_R = 0\) and \(C_V = \text{constant}\), Eqs. (10) can be solved analytically using simple exponential functions. For example, an initial particle number imbalance will decay exponentially with time.

Typically, starting with an initial particle number (temperature) imbalance \(\delta N_0\) \((\delta T_0)\) will lead to a time-dependent temperature (particle number) imbalance due to the coupling between particle and heat transport. As a consequence, the chemical potential imbalance \(\delta \mu = \mu_L - \mu_R\) and \(\delta \Delta = \Delta_L - \Delta_R\) will also depend on time. Eventually, as \(t \rightarrow \infty\), the system reaches an equilibrium state.

In the following we show and discuss three examples of such a time evolution displaying various quantities characterizing the system as a function of time. The time scale in Figs. 3–5 is fixed as follows: time can be expressed in units of \(\varepsilon_b/|\eta|^2\), where \(\varepsilon_b = \hbar^2/(2ma^2)\) and
FIG. 3: Time evolution of various quantities: (a) particle current ($I$) and heat current ($I_Q$), (b) superconducting gap in the left ($\Delta_L$) and right ($\Delta_R$) reservoir, (c) chemical potential difference ($\delta\mu$) and difference between gaps in the left and right reservoir ($\delta\Delta$), (d) particle number difference ($\delta N$) and temperature difference ($\delta T$). The sharp peak in the currents occurs for the time $t$ at which $|\delta\mu| = |\Delta_L - \Delta_R|$, i.e., when thermally excited quasiparticles are allowed to tunnel between the peaks in the DOS of the two reservoirs. The initial conditions chosen are $N = 2 \times 10^4$, $\Delta N_0/N = 0.04$, $T_L^0 = T_R^0 = T_0 = 0.07\varepsilon_b$, and $(k_{F_L,L0}^0)^{-1} = -1$.

$|\eta|^2 = |\eta_{kp}|^2$ is the modulus squared of the tunneling matrix element introduced after Eq. (3). As mentioned earlier, the time evolution of a system in the normal state within linear response corresponds to an exponential decay of the initial particle number imbalance. To get an order-of-magnitude estimate for the absolute time scale in seconds, we compare our results for the dimensionless linear response coefficient $1/\tau$ in $I = \delta N/\tau$, where the tilde denotes dimensionless quantities, with the experimental value $1/\tau_0 = 2.9\text{ s}^{-1}$ taken from Ref. 11. This leads to relation

$$\frac{\varepsilon_b\hbar}{|\eta|^2} = \tau_0/\tau.$$  

The time scale $\tau_0$ represents a characteristic particle transport time scale and is analogous to the RC-time of a capacitor circuit.

Figure 3 demonstrates a case in which a sharp peak in the current as a function of time appears. This can be understood in the semiconductor picture of the tunneling process: the BCS DOS at the edges of the gap, $E = \pm \Delta$, in both reservoirs is divergent, provided that both reservoirs are in the superfluid regime. Hence, if the condition $|\delta\mu(t)| = |\Delta_L(t) - \Delta_R(t)|$ is satisfied, electrons from a peak in the DOS of one reservoir are allowed to tunnel into the peak in the DOS of the other reservoir. This condition creates a logarithmic singularity in the integrals in Eqs. (8), (9) (in the absence of gap anisotropy and level broadening).

Moreover, a time-dependent temperature imbalance $\delta T(t)$ develops that exhibits a non-monotonic behavior and reaches its maximum value $\delta T_{\text{max}}$ at a certain time, see Fig. 3(d). The build-up of this temperature imbalance is a signature of the Peltier effect. For the case shown in Fig. 3 the initial conditions are chosen such that both reservoirs are in the superfluid regime throughout the time evolution: $N = 2 \times 10^4$, $\Delta N_0/N = 0.04$, $T_L^0 = T_R^0 = T_0 = 0.07\varepsilon_b$, and $(k_{F_L,L0}^0)^{-1} = -1$. The corresponding initial values of $T_{\text{c,L}}^0$ are $T_{\text{c,L}}^0 = 0.125\varepsilon_b$ and $T_{\text{c,R}}^0 = 0.119\varepsilon_b$.

In Fig. 4 we choose a negative initial particle number imbalance $\delta N_0/N = -0.04$ (while keeping $(k_{F,L}^0a)^{-1} = -1$) and an initial temperature $T_L^0 = T_R^0 = T_0 = 0.1248\varepsilon_b$ that lies between the initial transition temperatures of the two reservoirs. Since $T_{\text{c,L}}^0 = 0.119\varepsilon_b$ and $T_{\text{c,R}}^0 = 0.125\varepsilon_b$ in this case, the left reservoir is initially normal and the right one superfluid. During the time evolution, the left reservoir undergoes a transition to a superfluid state as shown in Fig. 4(b). Interestingly, this is not caused by lowering the temperature in the left reservoir. On the contrary, the temperature in the left reservoir actually temporarily rises. But the particle number (and hence the density) in the left reservoir rises which causes the transition from $\Delta_L = 0$ to $\Delta_L \neq 0$. As before, the calculation was done for $N = 2 \times 10^4$.

Figure 5 shows a more complex time evolution. The peaks in the current as a function of time appear for the same reason as in Fig. 3(a), but now the condition $|\delta\mu| = |\Delta_L(t) - \Delta_R(t)|$ is satisfied twice during the time evolution, see Fig. 5(c). The system also undergoes several superfluid transitions similar to Fig. 4(b). Finally, when the system equilibrates for $t \to \infty$, both reservoirs end up in the superfluid state. The initial conditions were chosen as $N = 2 \times 10^4$, $\Delta N_0/N = 0.04$, $T_L^0 = 0.132\varepsilon_b$, and...
T_R = 0.115 \varepsilon_b, T_0 = (T^0_L + T^0_R)/2, and (k_{F,L,a})^{-1} = -1.

As mentioned earlier, the induced temperature imbalance \( \delta T \) due to an initial particle number imbalance \( \delta N_0 \) is a signature of the Peltier effect. It shows a non-monotonic behavior as a function of time with a maximum \( \delta T_{\text{max}} \) at intermediate times, see Figs. 3(d) and 4(d). In Fig. 6 we show \( \delta T_{\text{max}} \) as a function of \( (k_{F,L,a})^{-1} \) for different values of the initial particle number imbalance \( \delta N_0 \) and initial temperature \( T^0_L = T^0_R = T_0 \). Each of the functions is divided into two sections monotonically increasing with increasing \( (k_{F,L,a})^{-1} \). The left section represents data from a system which is in the normal state, \( \Delta_{L,R}(t) = 0 \), during the whole time evolution, whereas for the right section \( \Delta_{L,R}(t) \neq 0 \), as in Fig. 3. Between the two sections, there is a “transient” regime, where superficial transitions occur, similar to the ones in Figs. 4 and 5. The generalized BCS theory\(^{20} \) on which our description of the BCS-BEC crossover is based is an approximation that becomes less accurate on approaching the unitary limit. However, the increase of the temperature imbalance starts already at relatively large (negative) values of \( (k_{F,L,a})^{-1} < -2 \). We therefore expect the trend to be qualitatively correct in a region approaching (but not too close) to unitarity. This increase of \( \delta T_{\text{max}} \) towards unitarity cannot be explained by particle-hole asymmetry alone but is due to a delicate interplay of the various factors in the integrands of Eqs. (8) and (9).

IV. CONCLUSION

To summarize, we have investigated particle and heat transport on the BCS side of the BCS-BEC crossover in a two-terminal setup with two reservoirs of interacting ultracold atoms. We have shown that a system initially out of equilibrium will show particle and/or thermal currents whose existence leads to characteristic time-dependent signatures, such as transitions between normal and superconducting states and resonant features in the currents as a function of time. An initial temperature imbalance can lead to a difference in chemical potentials at intermediate times. This is a signature of the Seebeck effect. Conversely, an initial particle number imbalance for two reservoirs at equal temperatures cannot lead to the build-up of a temperature difference at intermediate times, which is a signature of the Peltier effect. The maximal induced temperature imbalance increases if \( (k_{F,a})^{-1} \) moves closer to the unitarity limit.

In conclusion, our paper points out a variety of dynamical features visible in the equilibration process that can be used to pin-point the parameters of the system. An experimental confirmation of the Peltier effect discussed here is important since an additional cooling mechanism for ultracold fermionic atoms will be a valuable resource. Furthermore, transport experiments in systems of ultra-
cold atoms provide a fascinating laboratory in which the combination of particle and thermal currents can be explored in a regime that is not accessible to experiments with metallic superconductors.

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