

Improved position measurement of nanoelectromechanical systems using cross-correlations

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We consider position measurements using the cross-correlated output of two tunnel junction position detectors. Using a fully quantum treatment, we calculate the equation of motion for the density matrix of the coupled detector-detector-mechanical oscillator system. After discussing the presence of a bound on the peak-to-background ratio in a position measurement using a single detector, we show how one can use detector cross-correlations to overcome this bound. We analyze two different possible experimental realizations of the cross-correlation measurement and show that in both cases the maximum cross-correlated output is obtained when using twin detectors and applying equal bias to each tunnel junction. Furthermore, we show how the double-detector setup can be exploited to drastically reduce the added displacement noise of the oscillator.

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I. INTRODUCTION

It is expected that, in the near future, position measurements of nanomechanical systems will reach the quantum limit. Experimental progress in this direction is very fast and displacement sensitivities near the standard quantum limit have already been demonstrated¹⁻³.

In the current generation of experiments, the coupling between the resonator and the mesoscopic detector is typically very weak. The position measurement can therefore *not* be seen as a strong projective measurement. It is better described within the framework of weak measurement theory that was recently developed in the context of solid-state quantum computing⁴⁻⁷. This theory describes a continuous measurement process where the information about the measured object can be extracted, for instance, from the spectral density of the detector (and not simply from its average output). An important result in this theory is the Korotkov-Averin bound, which puts an upper limit of 4 to the ratio of the contribution of the measured state to the detector's spectral density, and the intrinsic background detector noise, for any linear detector measuring a two-level system.

Since a quantum position measurement by a mesoscopic detector can be described within the same theoretical framework as a qubit measurement, one might ask if such a bound also exists in the case of a position measurement. In this article, we first show that, for fixed system parameters, the peak-to-background ratio in the spectral density of a position detector weakly coupled to an oscillator is also bounded from above. This result is obtained by considering the example of a single tunnel-junction detector, a simple detector that has been thoroughly studied theoretically⁸⁻¹⁴ and realized experimentally^{15,16}.

Besides showing that the peak-to-background ratio is bounded in the typical single-detector position measurement, we also propose, in this article, two simple experimental configurations (Fig. 1) where, by using the cross-correlations between two detectors, the bound on the peak-to-background ratio can be overcome. As the

oscillator-independent parts of the output signal of the two detectors are uncorrelated, the background noise in these configurations is zero and therefore the peak-to-background ratio diverges. In the context of qubit readout, this idea has already been proposed in an insightful work by Jordan and Büttiker¹⁷ and was shown experimentally to improve readout fidelity¹⁸. Experimentally, position measurements should hence also profit from using cross-correlated detector outputs. We analyze in detail the two configurations presented in Fig. 1 and obtain analytical results for the optimal cross-correlated signal as a function of different detector parameters.

Previous studies¹⁹ of the position measurement problem focused on finding the conditions for quantum-limited detector sensitivity, under which one minimizes the total detector contribution to the output displacement noise. We show that the double-detector setup proposed here can in fact be used to almost totally get rid off the added displacement noise of the oscillator due to detector back-action. This is a remarkable result that nicely complements the general single-detector analysis made in Ref. 19.

The article is organized as follows: in Section II, we introduce the formalism used in the rest of the paper, viz., a master equation for the m -resolved density matrix, where m is the number of charges that have passed through the detector. This equation of motion allows us to find expressions for the combined moments of charge (detector) and oscillator quantities. In Section III, the formalism is applied to the case of one position detector coupled to the oscillator.¹² We analyze the peak-to-background ratio and show that this quantity is always bounded from above in the single-detector case. This bound cannot be made arbitrarily large simply by increasing the detector sensitivity. Section IV generalizes this treatment to a configuration with two detectors and demonstrates that measuring the current cross-correlations of the two detectors allows one to get arbitrarily high values of the peak-to-background ratio: i.e., it is possible to eliminate the bound that exist in the single-detector case. In Sec-

tion V, we demonstrate how the proposed setup can be used to diminish the added position noise of the oscillator induced by the presence of the detector, allowing position measurement beyond the standard quantum limit derived for a single detector.

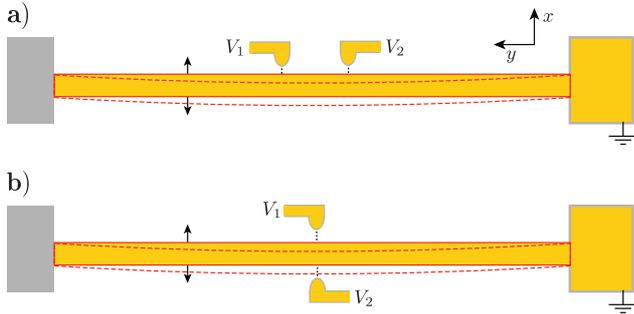


FIG. 1: (Color online) The two typical detector configurations examined in this article. In both cases, the movement of the oscillator is along the x direction in the xy plane, as depicted by the \leftrightarrow sign. a) *In-phase configuration*, where two detectors (with bias V_1 and V_2 , respectively) are located on the same side of the central part of the oscillator, such that both detectors couple in the same way to the position of the oscillator. This is covered in Sec. IV A. b) *Out-of-phase configuration*, where the detectors are located on each side of the oscillator. When the position of the oscillator is such that the tunneling amplitude of one junction is increased, the tunneling amplitude of the other junction is therefore decreased. This is covered in Sec. IV B.

II. EQUATION OF MOTION FOR THE DENSITY MATRIX

Approaches based on quantum master equations have proven useful in the study of nanomechanical systems^{20,21}. By writing the equation of motion for the density matrix of the full (detector and oscillator) system and tracing out the detector degrees of freedom, one can obtain an equation of motion for the reduced density matrix describing the evolution of the oscillator taking into account the coupling to the detector. In order to investigate electronic transport in the coupled system, it is useful to refine this approach to keep track of m , the number of charges that passed through the detector. This allows one to calculate an equation of motion for the m -resolved density-matrix^{12,13,22}, a quantum equivalent to the m -resolved master equation approach widely used in the study of transport properties of classical nanomechanical systems^{23,24}.

To study the current cross-correlations between two tunnel junction position detectors coupled to an oscillator, we use such a fully quantum approach. We label the detectors with the index $\alpha = 1, 2$ and model each of them as a pair of metallic leads with constant density of states Λ_α (in the energy range relevant to tunneling) coupled

via the tunneling Hamiltonian H_{tun} . The Hamiltonian for one detector can therefore be written as a sum of a bath Hamiltonian $H_{B,\alpha}$ describing the leads of junction α and a tunneling Hamiltonian $H_{\text{tun},\alpha}$

$$H_{\text{det},\alpha} = H_{B,\alpha} + H_{\text{tun},\alpha} \quad (1)$$

$$H_{B,\alpha} = \sum_k \varepsilon_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha} + \sum_q \varepsilon_{q,\alpha} c_{q,\alpha}^\dagger c_{q,\alpha} \quad (2)$$

$$H_{\text{tun},\alpha} = T_\alpha(\hat{x}) Y_\alpha^\dagger \sum_{k,q} c_{k,\alpha}^\dagger c_{q,\alpha} + T_\alpha(\hat{x}) Y_\alpha \sum_{k,q} c_{q,\alpha}^\dagger c_{k,\alpha}, \quad (3)$$

where $k(q)$ is a wave-vector in the right(left) lead. The coupling between the detector and the position of the oscillator is modeled by a linear x -dependence of the tunneling amplitude

$$T_\alpha(\hat{x}) = \frac{1}{2\pi\Lambda_\alpha} (\tau_{0,\alpha} + e^{i\eta_\alpha} \tau_{1,\alpha} \hat{x}). \quad (4)$$

In this equation, $\tau_{0,\alpha}$ is the bare (oscillator-independent) tunneling amplitude of detector α , \hat{x} is the position operator of the oscillator and $\tau_{1,\alpha}$ is the part of the full tunneling amplitude detector α that depends on the position of the oscillator. We allow for a general relative phase η_α , describing the details of the coupling between the tunnel junction and the oscillator. Such a phase can in principle be controlled by a magnetic flux penetrating an extended tunnel junction consisting of a loop containing two junctions, one of which couples to the oscillator. Note that in our notation $\tau_{0,\alpha}$ is dimensionless and $\tau_{1,\alpha}$ has dimensions of one over length and that we assume for simplicity that the tunneling amplitudes do not depend on the single particle energies $\varepsilon_{k,\alpha(q,\alpha)}$. The operator $Y_\alpha^{(\dagger)}$ decreases (increases) m_α , the number of charges that tunneled through junction α . Its presence in the tunneling Hamiltonian allows one to keep track of the transport processes that occur during the evolution of the system.

We are interested in calculating the equation of motion for the reduced, m_α -resolved, density matrix

$$\rho(m_1, m_2; t) = \langle m_1, m_2 | \rho_{\text{osc}} | m_1, m_2 \rangle, \quad (5)$$

where $\rho_{\text{osc}} = \text{Tr}_B\{\rho_{\text{tot}}\}$ is the reduced density matrix that is obtained by tracing out the leads' degrees of freedom from the full system density matrix. Within a Born-Markov approximation, the equation of motion of ρ_{osc} can be expressed as

$$\begin{aligned} \frac{\partial}{\partial t} \rho_{\text{osc}}(t) = & -\frac{i}{\hbar} [H_{\text{osc}}, \rho_{\text{osc}}(t)] \\ & - \frac{1}{\hbar^2} \int_{-\infty}^0 d\bar{t} \text{Tr}_B \{ [H_{\text{tun}}, [H_{\text{tun}}(\bar{t}), \rho_{\text{osc}}(t) \otimes \rho_B]] \}, \end{aligned} \quad (6)$$

where $H_{\text{tun}} = H_{\text{tun},1} + H_{\text{tun},2}$ is the total tunneling Hamiltonian, the trace is on both pairs of leads, ρ_B is

the coupled density matrix of the two sets of leads and

$$H_{\text{osc}} = \hbar\Omega(\hat{a}^\dagger\hat{a} + 1/2) = \frac{\hat{p}^2}{2M} + \frac{M\Omega^2\hat{x}^2}{2}, \quad (7)$$

$$H_{\text{tun}}(t) = \sum_{\alpha} e^{iH_{0,\alpha}t/\hbar} H_{\text{tun},\alpha} e^{-iH_{0,\alpha}t/\hbar}. \quad (8)$$

with $H_{0,\alpha} = H_{\text{osc}} + H_{B,\alpha}$. In our system, the Born approximation corresponds to assuming that tunneling in both tunnel junctions is weak enough so that it can be treated using second-order perturbation theory. The Markov approximation, on the other hand, is valid as long as the typical correlation times in the leads (\hbar/eV) are much shorter than $2\pi/\Omega$, i.e. the typical evolution time of the oscillator. In practice, this limits the applicability of the following results to the strongly biased case $eV \gg \hbar\Omega$. This is experimentally feasible since typical oscillator frequencies Ω are between 10 – 100 MHz and the measurements are done at a much larger bias voltage than these frequencies.^{1-3,16}

Since the leads of detector 1 are totally independent of those of detector 2, ρ_B can be written as a tensor product of the density matrices describing each pair of leads $\rho_B = \rho_{B_1} \otimes \rho_{B_2}$. Also, as $H_{\text{tun},\alpha}$ has no diagonal contribution in the basis that diagonalizes $H_{B,\alpha}$, the trace over leads α of a quantity that is linear in $H_{\text{tun},\alpha}$ vanishes. As a result of those two properties, the trace in Eq. (6) can be rewritten as a sum over two traces, each involving only one pair of leads

$$\begin{aligned} & \text{Tr}_B \{ [H_{\text{tun}}, [H_{\text{tun}}(\bar{t}), \rho_{\text{osc}}(t) \otimes \rho_B]] \} \\ &= \sum_{\alpha} \text{Tr}_{B_{\alpha}} \{ [H_{\text{tun},\alpha}, [H_{\text{tun},\alpha}(\bar{t}), \rho_{\text{osc}}(t) \otimes \rho_{B_{\alpha}}]] \}. \end{aligned} \quad (9)$$

This effectively makes the two-detector problem two single-detector problems. The trace over the leads' degrees of freedom is then carried out in the standard way.²⁵ As mentioned above, we are interested in calculating the time-evolution of the m_{α} -resolved density matrix. Thus, we have to calculate $\langle m_1, m_2 | \partial_t \rho_{\text{osc}} | m_1, m_2 \rangle$. We use the relations

$$\begin{aligned} \langle m_1, m_2 | Y_1 Y_1^\dagger \rho_{\text{osc}}(t) | m_1, m_2 \rangle &= \rho(m_1, m_2; t), \\ \langle m_1, m_2 | Y_1^\dagger \rho_{\text{osc}}(t) Y_1 | m_1, m_2 \rangle &= \rho(m_1 - 1, m_2; t), \\ \langle m_1, m_2 | Y_1^\dagger Y_1 \rho_{\text{osc}}(t) | m_1, m_2 \rangle &= \rho(m_1, m_2; t), \\ \langle m_1, m_2 | Y_1 \rho_{\text{osc}}(t) Y_1^\dagger | m_1, m_2 \rangle &= \rho(m_1 + 1, m_2; t), \end{aligned}$$

as well as the equivalent identities for detector 2 in Eq. (6) to find the equation of motion for $\rho(m_1, m_2; t)$.

Following a counting-statistics approach^{26,27}, it is particularly useful to express the equation of motion in terms of a counting field χ_{α} , the conjugate quantity to the transferred charge m_{α} . Indeed, Fourier-transforming in the transferred-charge indices m_{α} ,

$$\tilde{\rho}(\chi_1, \chi_2; t) = \sum_{m_1=-\infty}^{\infty} e^{i\chi_1 m_1} \sum_{m_2=-\infty}^{\infty} e^{i\chi_2 m_2} \rho(m_1, m_2; t) \quad (10)$$

leads to an equation of motion from which the time-dependence of all moments of m (for example, $\partial_t \langle m_{\alpha} \rangle, \partial_t \langle m_{\alpha}^2 \rangle, \dots$) can be determined. The current-current correlations can then be obtained by taking successive derivatives with respect to $(i\chi_{\alpha})$ of the equation of motion of $\tilde{\rho}(\chi_1, \chi_2; t)$.

In the regime of weak coupling between the oscillator and the detectors, we can write the equation of motion of $\tilde{\rho}(\chi_1, \chi_2; t)$ as

$$\begin{aligned} \frac{d}{dt} \tilde{\rho}(\chi_1, \chi_2; t) &= \frac{-i}{\hbar} [H_{\text{osc}}, \tilde{\rho}(t)] + \frac{i}{\hbar} \sum_{\alpha} [\bar{F}_{\alpha} \hat{x}, \tilde{\rho}(t)] \\ &- \frac{i}{\hbar} \sum_{\sigma,\alpha} \gamma_{\sigma,\alpha} [\hat{x}, \{\hat{p}, \tilde{\rho}(t)\}] - \frac{1}{\hbar^2} \sum_{\sigma,\alpha} D_{\sigma,\alpha} [\hat{x}, [\hat{x}, \tilde{\rho}(t)]] \\ &+ \sum_{\sigma,\alpha} \left(\frac{e^{i\sigma\chi_{\alpha}} - 1}{\tau_{1,\alpha}^2} \right) \times \\ &\left(\frac{2D_{\sigma,\alpha}}{\hbar^2} (\tau_{0,\alpha} + e^{i\sigma\eta_{\alpha}} \tau_{1,\alpha} \hat{x}) \tilde{\rho}(t) (\tau_{0,\alpha} + e^{-i\sigma\eta_{\alpha}} \tau_{1,\alpha} \hat{x}) \right. \\ &+ i \frac{\gamma_{\sigma,\alpha}}{\hbar} \left[\tau_{0,\alpha} \tau_{1,\alpha} (e^{i\sigma\eta_{\alpha}} \hat{p} \tilde{\rho}(t) - e^{-i\sigma\eta_{\alpha}} \tilde{\rho}(t) \hat{p}) \right. \\ &\left. \left. + \tau_{1,\alpha}^2 (\hat{p} \tilde{\rho}(t) \hat{x} - \hat{x} \tilde{\rho}(t) \hat{p}) \right] \right). \end{aligned} \quad (11)$$

Since $\tilde{\rho}(\chi_1 = 0, \chi_2 = 0; t) = \sum_{m_1} \sum_{m_2} \rho(m_1, m_2; t)$, taking $\chi_1 = \chi_2 = 0$ corresponds to completely tracing out the charge degrees of freedom. In this case, one finds that $\dot{\tilde{\rho}}(0, 0; t)$ is of Caldeira-Leggett form^{28,29}. We can thus identify the constants $D_{\sigma,\alpha}$ and $\gamma_{\sigma,\alpha}$ as, respectively, the diffusion and damping constants induced by forward ($\sigma = +$) or backward ($\sigma = -$) propagating currents in detector α . We can also identify \bar{F}_{α} as the average back-action force exerted on the oscillator by detector α . We find explicitly

$$D_{\sigma,\alpha} = \frac{\hbar^2}{4} \left(\frac{\tau_{1,\alpha}}{\tau_{0,\alpha}} \right)^2 [\Gamma_{\sigma,\alpha}(\hbar\Omega) + \Gamma_{\sigma,\alpha}(-\hbar\Omega)], \quad (12)$$

$$\gamma_{\sigma,\alpha} = \frac{\hbar}{2M\Omega} \left(\frac{\tau_{1,\alpha}}{\tau_{0,\alpha}} \right)^2 \left(\frac{\Gamma_{\sigma,\alpha}(\hbar\Omega) - \Gamma_{\sigma,\alpha}(-\hbar\Omega)}{2} \right), \quad (13)$$

$$\bar{F}_{\alpha} = \frac{1}{\hbar} \sin(\eta_{\alpha}) \left(\frac{\tau_{0,\alpha}}{\tau_{1,\alpha}} \right) \sum_{\sigma} 2\sigma D_{\sigma,\alpha}, \quad (14)$$

where the two inelastic tunneling rates are given by

$$\begin{aligned} \Gamma_{+,\alpha}(E) &= \frac{|\tau_{0,\alpha}|^2}{\hbar} \\ &\times \int_0^{\infty} d\varepsilon_{q,\alpha} f_{q,\alpha}(\varepsilon_{q,\alpha}) \left(1 - f_{k,\alpha}(\varepsilon_{q,\alpha} + E) \right), \end{aligned} \quad (15)$$

$$\begin{aligned} \Gamma_{-,\alpha}(E) &= \frac{|\tau_{0,\alpha}|^2}{\hbar} \\ &\times \int_0^{\infty} d\varepsilon_{k,\alpha} f_{k,\alpha}(\varepsilon_{k,\alpha}) \left(1 - f_{q,\alpha}(\varepsilon_{k,\alpha} + E) \right), \end{aligned} \quad (16)$$

involving a transfer of energy E from the oscillator to the lead electron. We denote by $\Gamma_{+,\alpha}$ the forward tunneling rate, i.e. the rate at which electrons tunnel in the direction favored by the voltage bias. The backward rate $\Gamma_{-,\alpha}$ corresponds to the reverse process. In Eqs. (15) and (16), $f_{k,\alpha} = f_{R,\alpha}(\epsilon_{k,\alpha})$ is the Fermi distribution function describing the local thermal equilibrium of the right lead of detector α and $f_{q,\alpha} = f_{L,\alpha}(\epsilon_{q,\alpha})$ is the same for the left lead.

Comparing these relations with the one derived in the single-detector case¹² shows that the full damping and diffusion coefficients governing the evolution of the oscillator are the sum of two single-detector contributions.

The Caldeira-Leggett form of Eq. (11) allows us to include the effect of direct coupling of the oscillator to the environment by adding detector-independent contributions $D_0 = 2M\gamma_0 k_B T_0$ and $\gamma_0 = \Omega/Q_0$ (where Q_0 is the extrinsic quality factor of the mode) to the previously derived diffusion and damping constants. The evolution of the oscillator is then governed by the two constants $D_{\text{tot}} = D_0 + \sum_{\sigma,\alpha} D_{\sigma,\alpha}$ and $\gamma_{\text{tot}} = \gamma_0 + \sum_{\sigma,\alpha} \gamma_{\sigma,\alpha}$. For the specific case where the electronic temperature is zero and where $eV_\alpha \gg \hbar\Omega$, current will only be possible along the ($\sigma = +$) direction, and both $\gamma_{-,\alpha}$ and $D_{-,\alpha}$ will be zero. In this case one can also show that $\gamma_{+,\alpha} = \hbar\tau_{1,\alpha}^2/(4\pi M)$ and that the diffusion parameters are given by $D_{+,\alpha} = M\gamma_{+,\alpha}eV_\alpha$.

The equation of motion for different moments $\langle x^j p^k \rangle$ of the oscillator can be evaluated by taking the trace of $x^j p^k \dot{\rho}(0, 0; t)$. More generally, equations of motion for combined moments of charge and oscillator quantities can be obtained by also considering derivatives with respect to the counting fields χ_α

$$\begin{aligned} & \frac{\partial}{\partial t} \langle x^{n_1} p^{n_2} m_1^{n_3} m_2^{n_4} \rangle \\ &= \text{Tr} x^{n_1} p^{n_2} \left(\frac{\partial^{(n_3+n_4)}}{\partial (i\chi_1)^{n_3} \partial (i\chi_2)^{n_4}} \dot{\rho}(\chi_1, \chi_2; t) \right)_{\chi_1=\chi_2=0}. \end{aligned} \quad (17)$$

III. SINGLE-DETECTOR CASE: BOUND ON THE PEAK-TO-BACKGROUND RATIO

One of the main motivations for studying position measurements using cross-correlated detector outputs is to remove the bound on the peak-to-background ratio that appears in the single-detector case, just like in the case of a weak measurement of a two-level system¹⁷. In this section, we first review the results of Clerk and Girvin (CG) [12] for the single-detector configuration, in the case where one considers the dc-biased, $T = 0$, tunnel junction where the x -dependent tunneling phase is $\eta = 0$. We then carefully analyze the peak-to-background ratio and show that this quantity is bounded from above in the single-detector case, for finite bias voltage and oscillator displacement.

Using the single-detector analogue of Eq. (11), CG showed that, under the conditions mentioned above and to first non-vanishing order in τ_1 , the current noise of a tunnel junction position detector is given by

$$S_I^{\text{tot}}(\omega) = 2e\langle I \rangle + \frac{e^3 V}{h} (2\tau_0 \tau_1)^2 \left(\frac{eV}{h} - \frac{\Omega}{4\pi} \frac{\Delta x_0^2}{\langle x^2 \rangle} \right) S_x(\omega), \quad (18)$$

where $\Delta x_0^2 = \hbar/(2M\Omega)$ is the average of x^2 in the ground state of the (quantum) harmonic oscillator and

$$S_x(\omega) = \frac{8\gamma_{\text{tot}}\Omega^2 \langle x^2 \rangle}{4\gamma_{\text{tot}}^2 \omega^2 + (\Omega^2 - \omega^2)^2} \quad (19)$$

its power spectrum. The full current noise is the sum of the usual frequency-independent Poissonian (shot) noise and the contribution of interest due to the coupling of the junction to the oscillator. This second part is itself expressed as the difference of a classical part (which is proportional to V^2) and a quantum correction (which is proportional to V).

A relevant figure of merit of such detectors is the peak-to-background ratio $\mathcal{R}(\omega)$: the ratio of the contribution of the oscillator to the full current noise at frequency ω over the unavoidable frequency-independent intrinsic detector noise. This ratio is maximal at $\omega = \Omega$ and, in the case where one only considers the $\propto V^2$ contribution in Eq. (18), was shown to be given by

$$\mathcal{R}(\Omega) = \frac{S_I^{\text{tot}}(\Omega) - 2e\langle I \rangle}{2e\langle I \rangle} = 4\tau_0^2 \frac{eV}{h\gamma_{\text{tot}}} \frac{\beta^2}{1 + \beta^2}, \quad (20)$$

where we used $\langle I \rangle = \langle \partial_t m_1(t) \rangle \simeq e^2 V \tau_0^2 (1 + \beta^2)/h$ and introduced the dimensionless sensitivity parameter $\beta^2 = \tau_1^2 \langle x^2 \rangle / \tau_0^2$.³⁰ At this point, one should proceed with care when maximizing \mathcal{R} with respect to the sensitivity parameter, as $\gamma_{\text{tot}} = \gamma_0 + \gamma_+$ depends on β through $\gamma_+ = (\Omega\tau_0^2/2\pi)(\Delta x_0^2/\langle x^2 \rangle)\beta^2$. Writing out explicitly all terms in \mathcal{R} that depend on β , one finds that

$$\mathcal{R}(\Omega) = \frac{2\tau_0^2}{\pi} Q_0 \frac{eV}{\hbar\Omega} \left(1 + Q_0 \frac{\tau_0^2}{2\pi} \frac{\Delta x_0^2}{\langle x^2 \rangle} \beta^2 \right)^{-1} \frac{\beta^2}{1 + \beta^2} \quad (21)$$

is a non-monotonic function of the sensitivity parameter β . For a given $\langle x^2 \rangle$, one can then find an optimal value

$$\beta_{\text{opt}}^4 = \frac{2\pi}{Q_0 \tau_0^2} \frac{\langle x^2 \rangle}{\Delta x_0^2}, \quad (22)$$

for which \mathcal{R} is maximal

$$\mathcal{R}_{\text{max}} = 4 \frac{Q_0 \tau_0^2}{2\pi} \left(\frac{eV}{\hbar\Omega} \right) \left(1 + \sqrt{\frac{Q_0 \tau_0^2}{2\pi} \frac{\Delta x_0^2}{\langle x^2 \rangle}} \right)^{-2}. \quad (23)$$

We can examine this result in two different limits. The first is when the damping is mainly detector-independent ($\gamma_0 \gg \gamma_+$), like in the case where the extrinsic quality

factor of the resonator is low, $Q_0 \ll \langle x^2 \rangle / (\tau_0^2 \Delta x_0^2)$. In this case, the maximal peak-to-background ratio,

$$\begin{aligned} \mathcal{R} &\simeq 4 \frac{\langle x^2 \rangle eV}{\Delta x_0^2 \hbar \Omega} \left(\frac{\tau_0^2 Q_0 \Delta x_0^2}{2\pi \langle x^2 \rangle} \right) \left(\frac{\beta^2}{1 + \beta^2} \right) \\ &\leq 4 \frac{\langle x^2 \rangle eV}{\Delta x_0^2 \hbar \Omega} \left(\frac{\tau_0^2 Q_0 \Delta x_0^2}{2\pi \langle x^2 \rangle} \right), \end{aligned} \quad (24)$$

is reached when the sensitivity parameter β is extremely large. However, since the rightmost term of Eq. (24) is by definition small in this limit, the peak-to-background ratio cannot become extremely large when the extrinsic resonator damping dominates the detector-induced one. Indeed, the real maximum of \mathcal{R} is reached when one considers the opposite limit of a very high resonator Q -factor¹², $Q_0 \gg \langle x^2 \rangle / (\tau_0^2 \Delta x_0^2)$. For $\gamma_0 = 0$, the peak-to-background ratio can be shown to obey

$$\mathcal{R} \simeq 4 \frac{\langle x^2 \rangle eV}{\Delta x_0^2 \hbar \Omega} \frac{1}{1 + \beta^2} \leq 4 \frac{\langle x^2 \rangle eV}{\Delta x_0^2 \hbar \Omega}. \quad (25)$$

In the single-detector case and for given system parameters (eV and $\langle x^2 \rangle$), the peak-to-background ratio is therefore always bounded whatever the strength of the coupling and the bound does not depend on Q_0 and τ_0 . As can be seen from Eq. (22), the peak-to-background ratio is in this second case maximal in the limit $\beta \rightarrow 0$ of vanishing coupling. While the optimal \mathcal{R} can be increased by increasing the bias voltage, we stress that our bound on \mathcal{R} denotes the optimal value of the peak-to-background reachable for a set of fixed system parameters.

The nature of the true bound on \mathcal{R} (i.e., the one found in the case $Q_0 \rightarrow \infty$) is very similar to the Korotkov-Averin bound that arises in the context of a weak measurements of a qubit. To make this more apparent, we can derive this bound following the linear-response approach that has been used to derive the bound on \mathcal{R} in the measurement of two-level systems, treating the detector as a position-to-current linear amplifier with responsivity (dimensionful gain) $\lambda = 2e^2 V \tau_0 \tau_1 / h$. As noted by CG, considering only the dominant $\propto V^2$ term in Eq. (18) corresponds to writing $\Delta S_I = S_I - 2e\langle I \rangle = \lambda^2 S_x(\omega)$. At resonance, the power spectrum $\Delta S_I = 2\lambda^2 \langle x^2 \rangle / \gamma$ is inversely proportional to the damping rate γ , in the same way that the response of the detector measuring a qubit is inversely proportional to the dephasing rate due to the measurement device. Moreover, in both cases one can show that the dephasing (damping rate) is proportional to the fluctuations of the bare input of the detectors. For a position detector in the high effective temperature limit $k_B T_{\text{eff}} \gg \hbar \Omega$, the detector-induced damping is indeed proportional to the symmetrized detector force noise¹⁹ $\gamma = \overline{S}_F / 2M k_B T_{\text{eff}}$, such that $\Delta S_I \leq 4M\lambda^2 \langle x^2 \rangle k_B T_{\text{eff}} / \overline{S}_F$. Also, since for a tunnel junction detector there is no reverse gain λ' and the real part of the cross-correlator $\overline{S}_{IF}(\omega)$ vanishes¹², the condition on quantum-limited efficiency of the position

measurement¹⁹

$$\overline{S}_I \overline{S}_F \geq \frac{\hbar^2}{4} (\text{Re}[\lambda - \lambda'])^2 + (\text{Re}[\overline{S}_{IF}])^2 \quad (26)$$

becomes exactly the one used to derive the Korotkov-Averin bound $\overline{S}_I \overline{S}_F \geq \frac{\hbar^2 \lambda^2}{4}$. We then find that $\mathcal{R} = \Delta S_I / S_I \leq 8 \langle x^2 \rangle k_B T_{\text{eff}} / (\hbar \Omega \Delta x_0^2)$. Using $k_B T_{\text{eff}} = eV/2$ in the tunnel junction system, this result corresponds exactly to Eq. (25), the bound previously derived using the equation-of-motion approach.

IV. PEAK-TO-BACKGROUND RATIO IN CURRENT CROSS-CORRELATIONS

Extending ideas from the qubit measurement problem¹⁷, we now demonstrate how to eliminate the bound on the peak-to-background ratio in a position measurement. Calculating the current-current correlations between two tunnel-junction position detectors, we show that for cross-correlation measurements, \mathcal{R} diverges. We also obtain analytical results for the cross-correlations in two typical cases.

To calculate the current cross-correlations, we use the generalized MacDonald formula^{31,32}, a general result (valid for stationary processes) that provides a way, in the present case, to relate the symmetrized cross-correlations to the Fourier sine-transform of the time-derivative of the covariance of m_1 and m_2 , the number of charges that tunneled through each junction. The generalized MacDonald formula reads

$$S_{I_1, I_2}(\omega) = 2e^2 \omega \int_0^\infty dt \sin(\omega t) K_{1,2}(t), \quad (27)$$

where we defined

$$K_{1,2}(t) = \left[\frac{d}{dt'} (\langle m_1 m_2 \rangle_{t'} - \langle m_1 \rangle_{t'} \langle m_2 \rangle_{t'}) \right]_{t'=t}. \quad (28)$$

In this last equation $\langle m_1 m_2 \rangle_t$ corresponds to $\text{Tr} m_1 m_2 \tilde{\rho}(0, 0, t)$ and represents the coupled moment of m_1 and m_2 at time t .

To proceed further, we restrict ourselves to the case of zero electronic temperature and dc-bias. In the following subsections, we analyze in detail the two different cases depicted schematically in Fig. 1. We have in mind that a realization of the setup shown in Fig. 1 is made in a similar way as the single-detector setup in Ref. 16. This means that the tunnel junctions correspond to atomic point contacts (formed by electromigration) which are separated by about 1 nm from the oscillator. In contrast, the two detectors are assumed to be separated from each other by at least 20 nm. Therefore, capacitive cross-talking between the detectors will play a negligible role.³³

A. In-phase configuration

We will first consider the case where both $\eta_1 = \eta_2 = 0$, the case where both tunnel junctions are located on the

same side of the oscillator, *cf.* Fig. 1 a). To calculate the cross-correlations, we use Eq. (17) (with $n_1 = n_2 = 0$), to find that

$$\begin{aligned} \frac{d}{dt}\langle m_\alpha \rangle_t &= \frac{eV_\alpha}{h} (\tau_{0,\alpha}^2 + 2\tau_{0,\alpha}\tau_{1,\alpha}\langle x \rangle + \tau_{1,\alpha}^2 \langle x^2 \rangle) - \gamma_{+,\alpha}, \\ \frac{d}{dt}\langle m_1 m_2 \rangle_t &= \\ &\frac{eV_1}{h} (\tau_{0,1}^2 \langle m_2 \rangle_t + 2\tau_{0,1}\tau_{1,1}\langle x m_2 \rangle_t + \tau_{1,1}^2 \langle x^2 m_2 \rangle_t) \\ &+ \frac{eV_2}{h} (\tau_{0,2}^2 \langle m_1 \rangle_t + 2\tau_{0,2}\tau_{1,2}\langle x m_1 \rangle_t + \tau_{1,2}^2 \langle x^2 m_1 \rangle_t) \\ &- \gamma_{+,1}\langle m_2 \rangle_t - \gamma_{+,2}\langle m_1 \rangle_t, \end{aligned}$$

and therefore that $K_{1,2}(t)$ in this case is given by

$$\begin{aligned} K_{1,2}(t) &= 2\frac{eV_1}{h}\tau_{0,1}\tau_{1,1}\langle\langle x m_2 \rangle\rangle_t + \frac{eV_1}{h}\tau_{1,1}^2\langle\langle x^2 m_2 \rangle\rangle_t \\ &+ 2\frac{eV_2}{h}\tau_{0,2}\tau_{1,2}\langle\langle x m_1 \rangle\rangle_t + \frac{eV_2}{h}\tau_{1,2}^2\langle\langle x^2 m_1 \rangle\rangle_t. \end{aligned} \quad (29)$$

where the double bracket denotes the covariance of two quantities: $\langle\langle ab \rangle\rangle_t \equiv \langle ab \rangle_t - \langle a \rangle_t \langle b \rangle_t$. This means that, to lowest order in $\tau_{1,\alpha}$, the *full* cross-correlated output of the detectors is given in this configuration by

$$\begin{aligned} S_{I_1, I_2}(\omega) \Big|_{\substack{\eta_1=0 \\ \eta_2=0}} &= 4e^2\omega \int_0^\infty dt \sin(\omega t) \times \\ &\left(\frac{eV_1}{h}\tau_{0,1}\tau_{1,1}\langle\langle x m_2 \rangle\rangle_t + \frac{eV_2}{h}\tau_{0,2}\tau_{1,2}\langle\langle x m_1 \rangle\rangle_t \right). \end{aligned} \quad (30)$$

The cross-correlated signal does not contain any oscillator-independent contribution. Using Eq. (17), a

closed system of differential equations involving $\langle\langle p m_\alpha \rangle\rangle_t$ and $\langle\langle x m_\alpha \rangle\rangle_t$ can be generated. This system can be solved, using the boundary conditions $m_\alpha(0) = 0$ and assuming that all averages that do not contain m_α are time-independent and can therefore be evaluated in the stationary ($t \rightarrow \infty$) limit²³.

Solving for the different covariances, we find that the current cross-correlations can be written as

$$\begin{aligned} S_{I_1 I_2}^{\text{tot}}(\omega) &= e^2(2\tau_{0,1}\tau_{0,2})(2\tau_{1,1}\tau_{1,2}) \\ &\left(\frac{e^2 V_1 V_2}{h^2} - \frac{e(V_1 + V_2)}{2h} \frac{\Omega}{4\pi} \frac{\Delta x_0^2}{\langle x^2 \rangle} \right) S_x(\omega), \end{aligned} \quad (31)$$

$$= \lambda_1 \lambda_2 \left(1 - \frac{\hbar\Omega(eV_1 + eV_2)}{4eV_1 eV_2} \frac{\Delta x_0^2}{\langle x^2 \rangle} \right) S_x(\omega), \quad (32)$$

where we introduced the gains $\lambda_\alpha = 2e^2\tau_{0,\alpha}\tau_{1,\alpha}V_\alpha \cos(\eta_\alpha)/h$. Evidently, the cross-correlated output of the detectors (31) does not contain any frequency-independent background noise. The peak-to-background ratio $\mathcal{R}(\Omega)$ therefore diverges for all values of γ_0/γ_+ , not because of an increased signal but due to the absence of background noise in this configuration.

For this type of measurement, a relevant figure of merit of the detection system \mathcal{R}_c is the ratio of the cross-correlated output over the frequency-independent noise power of individual detectors: $\mathcal{R}_c = S_{I_1, I_2}^{\text{tot}}(\Omega)/\sqrt{S_1 S_2}$, where $S_\alpha = 2e\langle I_\alpha \rangle$. For our position detector, we find

$$\mathcal{R}_c = \frac{|S_{I_1, I_2}^{\text{tot}}|}{\sqrt{S_1 S_2}} = \frac{4}{1 + \frac{\gamma_0}{\gamma_+}} \frac{1}{\sqrt{(1 + \beta_1^2)(1 + \beta_2^2)}} \frac{\tau_{1,1}\tau_{1,2}}{\tau_{1,1}^2 + \tau_{1,2}^2} \frac{\sqrt{V_1 V_2}}{V_1 + V_2} \frac{e(V_1 + V_2)}{\hbar\Omega} \frac{\langle x^2 \rangle}{\Delta x_0^2} \leq \frac{e(V_1 + V_2)}{\hbar\Omega} \frac{\langle x^2 \rangle}{\Delta x_0^2}, \quad (33)$$

where we used $2xy \leq (x^2 + y^2)$. From this inequality, we see that the maximal cross-correlated output is found for (i) twin-detectors (where $\tau_{1,1} = \tau_{1,2}$) and (ii) equal bias voltages $V_1 = V_2$. Also, like in the single-detector case, \mathcal{R}_c is maximal in the limit where there is no extrinsic oscillator damping γ_0 and where the correction to the average current due to the coupling to the oscillator vanishes ($\beta_\alpha \rightarrow 0$).

Once again it is instructive to compare our value of \mathcal{R}_c for twin detectors with the equivalent result in the case of a weak measurement of a qubit using cross-correlations¹⁷. In the latter case, the cross-correlated output was shown to be limited to 1/2 of the single-detector signal due to the increased (doubled) detector-induced dephasing. This is the same here.

B. Out-of-phase detection

We can also analyze the case where one detector couples to $+x$ and the other to $-x$, as would happen if the two detectors were located on opposite sides of the resonator (see Fig. 1). In terms of the tunneling phases η_α , this corresponds to taking $\eta_1 = 0$ and $\eta_2 = \pi$. Using Eq. (27), the cross-correlations are then given by

$$\begin{aligned} S_{I_1, I_2}(\omega) \Big|_{\substack{\eta_1=0 \\ \eta_2=\pi}} &= 4e^2\omega \int_0^\infty dt \sin(\omega t) \times \\ &\left(\frac{eV_1}{h}\tau_{0,1}\tau_{1,1}\langle\langle x m_2 \rangle\rangle_t - \frac{eV_2}{h}\tau_{0,2}\tau_{1,2}\langle\langle x m_1 \rangle\rangle_t \right). \end{aligned} \quad (34)$$

As the coupling between detector 1 and the oscillator is the same as in the previous case $\langle\langle xm_1 \rangle\rangle_t$ remains unchanged in this second configuration. The covariance $\langle\langle xm_2 \rangle\rangle_t$ on the other hand changes sign (but keeps the same norm) in this new configuration. Equation (34) then yields

$$S_{I_1, I_2}(\omega) \Big|_{\substack{\eta_1=0 \\ \eta_2=\pi}} = -S_{I_1, I_2}(\omega) \Big|_{\substack{\eta_1=0 \\ \eta_2=0}}. \quad (35)$$

The cross-correlations in the second configuration are the same as in the first one, but of negative sign. From an amplifier point of view, this is easily explained since putting $\eta_2 = \pi$ corresponds to transforming $\lambda_2 \rightarrow -\lambda_2$ in $S_{I_1, I_2} \simeq \lambda_1 \lambda_2 S_x$. Finally, note that this configuration was analyzed for two single-electron transistor position detectors coupled to a classical oscillator, in Ref. [31] by Rodrigues and Armour. In their article, these authors only explicitly calculated zero-frequency cross-correlations between the currents in both detectors, but they conjectured that, at the resonance frequency of the oscillator, this detector-configuration (corresponding to $\eta_1 = 0, \eta_2 = \pi$ in our approach) should yield strong negative cross-correlations, just like the ones predicted here.

V. BOUND ON THE ADDED DISPLACEMENT NOISE

As shown in Sec. III, to derive the equivalent of the Korotkov-Averin bound in a position measurement, one needs to consider the *full* current noise, where no distinction is made between the signal due to the intrinsic equilibrium fluctuations of the oscillator $S_I^{\text{eq}}(\omega)$ and the remainder of the signal $S_I^{\text{add}}(\omega)$. This second contribution contains, amongst other things, the added signal due to heating of the oscillator by the detector. When trying to measure precisely the equilibrium fluctuations of a nanomechanical oscillator however, it is important to consider the two contributions separately: $S_I^{\text{eq}}(\omega)$ is exactly what you would like to measure while $S_I^{\text{add}}(\omega)$ limits the sensitivity of the measurement. When using a single linear detector like the tunnel junction, this measurement sensitivity is quantum-mechanically bounded from below¹⁹.

When discussing this bound on added noise, one usually considers the added *displacement* noise, that corresponds to the added current noise referred back to the oscillator. We therefore introduce the total displacement noise S_x^{tot} , defined as

$$S_x^{\text{tot}}(\omega) = \frac{S_I^{\text{tot}}(\omega)}{\lambda^2} = S_x^{\text{add}}(\omega) + S_x^{\text{eq}}(\omega), \quad (36)$$

where λ is the x -to- I gain of the detector, $S_x^{\text{add}}(\omega)$ is the part of the full displacement spectrum that arises due to the presence of the detector. In the relevant limit of a detector with a high power gain ($eV \gg \hbar\Omega$), it was shown using general arguments that $S_x^{\text{add}}(\Omega) \geq \hbar/M\Omega\gamma_{\text{tot}}$: the

best possible detector therefore adds exactly as much noise as a zero-temperature bath of frequency Ω ^{19,34}.

Before discussing the limit on the added displacement noise in a cross-correlation setup, it is helpful to describe how the quantum limit on $S_x^{\text{add}}(\Omega)$ is reached in a single-detector configuration. Let's consider for definitiveness the experimentally relevant configuration where $eV \gg k_B T_0 > \hbar\Omega$. For a measurement to be quantum limited, the effective temperature of the oscillator $T_{\text{eff}} = (\gamma_+ eV/2 + \gamma_0 k_B T_0)/(k_B \gamma_{\text{tot}})$ must not be dramatically higher than T_0 . This is natural, since added fluctuations due to the higher effective temperature are, by definition, unwanted back-action noise. In this regime, one therefore cannot expect S_x^{add} to be close to the quantum limit unless $\gamma_+ \ll \gamma_0$. The regime of γ_+/γ_0 in which quantum-limited displacement sensitivity can be achieved is therefore very different from the one where the bound on the peak-to-background ratio can be reached. Using the expression for the full current noise derived earlier (Eq. (18)), we write the full position noise as

$$S_x^{\text{tot}}(\omega) = \frac{S_I^{\text{tot}}(\omega)}{\lambda^2} = \frac{2e\langle I \rangle}{\lambda^2} + \left(1 - \frac{\hbar\Omega}{2eV} \frac{\Delta x_0^2}{\langle x^2 \rangle}\right) S_x(\omega), \quad (37)$$

$$= \frac{2e\langle I \rangle}{\lambda^2} + 8M\gamma_{\text{tot}}k_B T_{\text{eff}}|g(\omega)|^2 - 2M\gamma_{\text{tot}} \frac{(\hbar\Omega)^2}{eV}|g(\omega)|^2, \quad (38)$$

where in the last line we introduced the oscillator's response function $g^{-1}(\omega) = M[(\Omega^2 - \omega^2) + 2i\gamma_{\text{tot}}\omega]$. Splitting the second term into a detector dependent and independent part, we find

$$S_x^{\text{eq}} = 8M\gamma_0 k_B T_0 |g(\omega)|^2, \quad (39)$$

$$S_x^{\text{add}} = \frac{2e\langle I \rangle}{\lambda^2} + 8M\gamma_+ \frac{eV}{2}|g(\omega)|^2 - 2M\gamma_{\text{tot}} \frac{(\hbar\Omega)^2}{eV}|g(\omega)|^2. \quad (40)$$

This way of writing the equilibrium fluctuations implies that we consider $\gamma_{\text{tot}} \simeq \gamma_0$ in $g(\omega)$, in agreement with our previous assumption that $\gamma_+ \ll \gamma_0$. The added noise contains three contributions, corresponding to the detector shot noise, the detector-induced heating of the oscillator and a correction ($\propto \hbar\Omega/eV$) arising from the cross-correlation between the detector output noise and the back-action force, \overline{S}_{IF} , respectively. Explicitly, taking $\langle I \rangle \simeq e^2 \tau_0^2 V/h$,³⁵ we obtain

$$S_x^{\text{add}} = \frac{\pi\hbar}{eV\tau_1^2} + \frac{\hbar\tau_1^2 eV |g(\omega)|^2}{\pi} - 2M\gamma_{\text{tot}} \frac{(\hbar\Omega)^2}{eV}|g(\omega)|^2. \quad (41)$$

For a fixed bias voltage, the relevant tunable parameter is directly the detector-oscillator coupling τ_1 (and not the dimensionless sensitivity parameter β , since S_x^{add} is independent of τ_0).³⁶ For strong coupling, S_x^{add} is dominated by heating of the oscillator, while for weak coupling, the shot noise contribution ($\propto 1/\tau_1^2$) dominates.

This is the regime in which the current generation of experiments are operated¹⁶. There is an optimal coupling $\tau_{1,\text{opt}}^2 = \pi/(eV|g(\omega)|)$ that minimizes the total added noise. At the resonance, we recover the inequality

$$S_x^{\text{add}}(\Omega) \geq \left(1 - \frac{\hbar\Omega}{2eV}\right) \frac{\hbar}{\gamma_{\text{tot}}M\Omega}, \quad (42)$$

where the bound is reached when $\tau_1 = \tau_{1,\text{opt}}$. This is the quantum limit on the added displacement noise for the single-detector configuration. In passing, we note that the effective temperature of the oscillator when the coupling strength τ_1 is optimal is

$$T_{\text{eff}} = T_0 + \frac{\hbar\Omega}{4k_B}, \quad (43)$$

in agreement with the general analysis of Ref. [19]. The heating of the oscillator by the detector is, as expected, very low when doing a quantum-limited measurement. We can now show how cross-correlations can be used to beat the quantum limit on S_x^{add} derived in the single-detector case. In both cross-correlation configurations ($\eta = 0, \pi$), $S_x^{\text{tot}} = S_{I_1, I_2}/\lambda_1\lambda_2$ is identical. Like in the single-detector case, we separate the total position fluctuations in two parts

$$S_x^{\text{eq}} = 8M\gamma_0k_B T_0 |g(\omega)|^2, \quad (44)$$

$$\frac{S_x^{\text{add}}}{M|g(\omega)|^2} = 4 \left(\sum_{\alpha} \gamma_{+, \alpha} eV_{\alpha} \right) - \gamma_{\text{tot}} \frac{(\hbar\Omega)^2 (eV_1 + eV_2)}{eV_1 eV_2}. \quad (45)$$

The cross-correlated position spectrum does not contain the frequency-independent shot noise contribution that diverges for low coupling ($\propto 1/\tau_1^2$). Therefore, one does not need to tune the coupling to equilibrate the “shot noise” and back-action “heating” contributions. Instead, one can freely tune the couplings $\tau_{1,\alpha}$ such that $S_x^{\text{add}}(\omega)$ vanishes completely. We find $S_x^{\text{add}} = 0$ for $\tau_{1,\alpha,\text{opt}}^2 = 4\pi M\gamma_{+,\alpha,\text{opt}}/\hbar$, where

$$\gamma_{+,\alpha,\text{opt}} = \frac{\gamma_{\text{tot}}}{4} \left(\frac{\hbar\Omega}{eV_{\alpha}} \right)^2. \quad (46)$$

At the optimal coupling point, the effective temperature

of the oscillator is

$$T_{\text{eff}} = T_0 + \left(\frac{\hbar\Omega}{eV_1} + \frac{\hbar\Omega}{eV_2} \right) \frac{\hbar\Omega}{8k_B}. \quad (47)$$

In the regime of interest ($eV_{\alpha} \gg \hbar\Omega$), the additional heating of the oscillator considerably reduced from the single-detector value.

VI. CONCLUSION

In this article, we have shown that, for a tunnel-junction position detector coupled to a nanomechanical oscillator, the optimal peak-to-background ratio \mathcal{R} at the resonance frequency of the oscillator is bounded. In contrast to the universal (independent of all system parameters) bound derived for a continuous weak measurement of qubits ($\mathcal{R} \leq 4$), the new bound derived for position measurements is a function of the effective temperature of the oscillator and its average displacement. We have also shown that adding a second detector and using the cross-correlations between the two detectors allows one to eliminate this bound on \mathcal{R} . We have analyzed in detail the cross-correlated output of the position detectors in two typical configurations, and have shown that in both cases the optimal cross-correlated signal is measured by twin detectors. We also investigated the quantum-limit on the added displacement noise and shown that it is possible to totally eliminate the added displacement noise by doing a cross-correlated measurement. This configuration therefore opens the door for displacement measurement with sensitivities better than the quantum limit.

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